Role of Parity Violating Hamiltonian In-Non-Relativistic Quark Model for Non-Leptonic Hyperon Weak Transitions

by

Jasem Al-Alawi

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
Requirements for the Degree of

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ACKNOWLEDGMENTS

I am deeply obliged to Professor Riazuddin for his supervision and guidance in this research. I also wish to thank him for his advice and help all along my studies at K.F.U.P.M.. Special gratitude is due to my committee members: Professor H. A. Mavromatis and Dr. N. Zettilli for their cooperation and valuable remarks.

My thanks is also due to K.F.U.P.M., represented by the chairman and faculty of the physics department for their support.

Finally, I wish to thank all my colleagues and friends for their assistance and support throughout my studies at K.F.U.P.M.

TABLE OF CONTENTS

LIST OF TABLES

FIGURES CAPTIONS

THESIS ABSTRACT

NAME OF STUDENT : AL-ALAWI, JASEM

TITLE OF STUDY : ROLE OF PARITY VIOLATING HAMILTONIAN IN NON-RELATIVISIC QUARK MODEL FOR NON-LEPTONIC HYPERON WEAK TRANSITIONS.

MAJOR FIELD : PHYSICS

DATE OF DEGREE : JANUARY, 1996

A non-relativistic quark model has been used in the calculation of the K^*-pole contributions to s-wave amplitudes for non-leptonic decays of hyperons, which involve $\overline{K}^{*0} - \pi^0$ and $K^{*-} - \pi^-$ weak transitions. Such corrections were thought to be one way of removing the long standing discrepancy between s- and p- wave amplitudes obtained in the current-algebra approach to non-leptonic decays of hyperons. Although the calculation of the K^* contribution on the quark model is found to be negligible, it gives the right order of magnitude of small observed deviations from the $\Delta I=1/2$ rule. A comparison of the observed deviation from $\Delta I=1/2$ rule for s-wave hyperon decay amplitudes with estimated ones is discussed.

MASTER OF SCIENCE

KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS

Dhahran , Saudi Arabia

January, 1996

خلاصة الرسالة

اسم الطالب الكامل : جاسم حسن العلوي

عنوان الاطروحـــة : وظيفة الهميلتونين المنتهك للتماثل في نموذج كوارك الغير نسبي لجسيمات الهيبرونات ذات التحولات الضعيفه والغير لبتونيه

التخـــــــصص : فيزياء

تـاريخ الشهـــادة : يناير ١٩٩٦

تم استعمال نموذج كوارك الغير نسبي في حساب اضافة الـ K^* الى سـعات موجـات س لـلا نحلات الغير لبتونيه للهيبرونات، التي تتضمنت التحولات الضعيفه لـ $\pi^0 - \overline{K}^{*0}$ و $K^{*-} - \pi^-$.
كان يعتقد ان هذه التصحيحات تمثل احدى الطرق لازالة التناقض الذي ظـل قائمـا لفـترة طويلـة بين سعات موجات س و ب والتي حصل عليها باستخدام طريقـة الجبر الحديـث الى الانحلالات الغير لبتونية للهيبرونات. و برغم من ان حساب اضافة الـ K^* في نموذج كوارك وجد انـه مهمـلا الا انه اعطى السعة الصحيحة لـلانحرافات الصغيرة المشاهدة عن قانون $\Delta I=1/2$. ثم اجرينـا مقارنة بين الانحراف المشاهد من خلال التجربة والمقدر من خلال الحسابات عن قانون $\Delta I=1/2$ لسعات موجات س المنحلة من الهيبرونات.

درجة الماجستير في العلوم
جامعة الملك فهد للبترول والمعادن
الظهران ، المملكة العربية السعودية
يناير ١٩٩٦

CHAPTER 1

INTRODUCTION

The hyperon particles experience several non-leptonic weak decays, all of which have been observed. From the conservation of angular momentum, these decays involve both the s- and p-waves, where the s-waves arise from the parity violating part of the Hamiltonian and the p-waves arise from the parity conserving part. The current-current form of the weak interactions by itself dose not guarantee the dominance of $\Delta I=1/2$ rule over $\Delta I=3/2$ observed experimentally. The current algebra approach together with the partial conservation of axial vector current (PCAC) developed[2] in the 1960's was very successful in dealing with the processes which involve soft pions i.e. where $|q_\pi|/M \ll 1$, q_π being the momentum of the pion and M being typically the nucleon mass. The current algebra approach applied to the non-leptonic decays of hyperons, where the $|q_\pi|/M \ll 1$ is satisfied, using the current-current form of weak interaction Hamiltonian, was quite successful in reproducing the gross featuers of these decays[1]. In this approach, both the s- wave and p- wave amplitudes are determined by matrix elements of the form $\left\langle B_s \middle| H_W^{pc} \middle| B_r \right\rangle$, where B_r and B_s denote ordinary baryons. The s-wave amplitudes are determined by the current commutator term in the soft pion limit where the above matrix elements, while for the p- wave amplitudes the above matrix elements enter through the baryon-pole approximation. Thus the question $\Delta I=1/2$ rule for non-leptonic hyperon decays as well as that of scale hinge on the octet dominance of the matrix elements $\langle B_s | H_w^{pc} | B_r \rangle$ and on their scale.

It has been shown[3-5] that the current-current picture involving ordinary (V-A) charged currents, the quark-quark scattering diagram through $W^{\pm}$ exchange, in the leading non-relativistic limit, gives rise to:

i) $\Delta I=1/2$ or octet rule for the matrix elements $\langle B_s | H_w^{pc} | B_r \rangle$

ii) D/F ratio for the SU(3) parametrization of the above matrix matrix elements to be -1; this parametrization is defined as

$$\langle B_s | H_w^{pc} | B_r \rangle = \bar{u}\sqrt{2}[2Fif_{6rs} + 2Dd_{6rs}]u \tag{1.1}$$

where $\bar{u}$ and u are Dirac spinors and f_{ijk} and d_{ijk} are the usual structrue constants for SU(3).

iii) the right order of magnitude of the matrix elements $\langle B_s | H_w^{pc} | B_r \rangle$.

One may mention that the non-relativistic quark model has been quite sucessful in explaining properties of hadrons as well as their mass spectra and magnetic moments[6]. Since the matrix elements $\langle B_s | H_w^{pc} | B_r \rangle$ are similar to $\langle B_s | H_{Mass} | B_r \rangle$, it may not be a bad idea to apply the non-relativistic quark model to the calculation of $\langle B_s | H_w | B_r \rangle$, which in fact has given encourging results as mentioned above and further discussed below.

Now it is well known[7] that p-wave amplitudes for non-leptonic hyperon decays are well fitted by

$$D/F \approx -0.85 \ ,$$
$$F \approx 4.7 \times 10^{-5}\ \mathrm{MeV} \quad , \qquad (1.2)$$

to be compared with the quark model values[3-5] of D/F= -1 and F≈3.5×10^{-5} MeV, so that this model works within 15 to 25 %. However, with the values in (1.2) the s-wave amplitudes as given by the commutator term alone in the soft pion limit would come out to be larger by about a factor of 2. In fact, s-wave amplitudes as given by the commutator term alone require D/F ratio to be about -0.4. There are, however, important corrections to the commutator term in the soft pion limit, the inclusion of which would hopefully reduce the s-wave amplitudes to their experimental values with the parameters given in (1.2). We now come to the question of the corrections to the commutator term for the s-wave amplitudes. Such corrections, which have been studied previously, are of three types (each involving H_W^{pv}), are SU(3) violating, and each can be adjusted so as to cancel in part the commutator contribution and thus bring the s-wave amplitudes with parameters in Eq.(1.2) in agreement with experiment. One type is the K^* pole contribution considered in Ref. 7, where the essential parameter involved in this contribution is fitted instead of being calculated. However, we shall show that if this contribution is computed in the quark model to be consistent with the approach being followed here, it turns out to be too small to be relevant. This contribution also violates the $\Delta I=1/2$ rule, but a positive out come of our calculation is that it gives the right order of magnitude of the observed small deviations from the $\Delta I=1/2$ rule. Another type is from decuplet poles[8]. Again there is the question of scale, i.e., how big is this contribution if instead of fitting it, it is actually calculated. The third type[9] appears to be more promising and is in the spirit of quark model considerations. It comes from the first negative parity level of the baryon spectrum $(70,1^{-1})$ which are connected to ground state in the quark model approach by the parity-violating part of H_W. These corrections are of order[9] $\delta m/\omega$ where δm is the SU(3) breaking parameter ~ 200MeV while ω is the baryon level spacing ≈ 400-500 MeV. These corrections satisfy the $\Delta I=1/2$ rule and in part the too large contribution from the commutator, thus bringing the s-wave amplitudes also in agreement with experiment when the parameters of Eq.(1.2) are used.

The plan of the thesis is as follows: The second chapter places the weak interaction Hamiltonian in its framework in particle physics and briefly summarizes all types of weak decays and provides the general features that can describe the non-leptonic or hadronic processes such as the $\Delta I=1/2$ or octet rule where several important relations of s- and p- wave amplitudes have been derived . Also , the chapter reviews the application of current algebra and PCAC on hadronic processes and supplies the most recent experimental data for s- and p- wave amplitudes . The third chapter describes baryons and mesons on the quark model and also explains the non-relativistic quark model for non-leptonic hyperon decays where we will see the ability of such model to recover not only the $\Delta I=1/2$ rule but also the right order of magnitude of the scale required to reproduce the fit for s- and p-wave amplitudes. The chapter states clearly the discrepancy between the quark model predictions and the experimental results of the s- and p- wave amplitudes and gives the two possible contributions to s-wave amplitudes namely the K^*-pole contribution and the contribution from the first negative parity level of the baryon spectrum $(70,1^-)$. The fourth chapter specifies the need of calculating the K^*-pole contribution in the quark model and displays our results for the two selected weak transitions . It is shown that the K^*-pole contribution in the quark model is too small to be of relevance. But this contribution as calculated violates the $\Delta I=1/2$ rule and a comparison between the observed deviation from $\Delta I=1/2$ rule for s-wave hyperon decay amplitude estimated for K^*-π transition is depicted. Finally , in chapter five we summerize our conclusions which have been derived from the results we obtained in the previous chapter . The appendices A and B provide the necessary information on how we have proceeded with our calculations .

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CHAPTER 2

Brief Review of Weak Interaction

Particle physics is the study of the ultimate constituents of matter and the interactions between them. A few decades ago, matter was thought to be composed only of a few particles - the proton, neutron, electron and neutrino, together with the mediator of their electromagnetic interactions - the photon. Now, matter, according to the present evidence, is believed to be composed of hundreds of particles, all of which are built from two types of fundamental spin- 1/2 fermions called quarks and leptons[1].

Particles can be classified into two major groups: the strongby interacting particles or hadrons and leptons which do not take part in strong interaction[1,2,3].

2.1 Leptons

The lepton family consists of six known spin-1/2 fermions. The leptons occur in distinct doublets or generation[4], as follows

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$$

The three charged leptons (electron e^-, muon μ^- and tau lepton τ^-) have charge Q = -e and experience electromagnetic and weak interactions, while the neutrinos (the upper member of the doublet), which possess zero electric charge, are distinguished by having only weak interactions with other particles. For each lepton there is a corresponding antilepton with the same mass : e^+, μ^+, τ^+, $\bar{\nu}_e$, $\bar{\nu}_\mu$, $\bar{\nu}_\tau$ see table 2.1. A conserved quantum number called lepton number is associated with each generation of leptons. In particular the number (L_e, L_μ, L_τ) is +1 for the lepton indicated and -1 for the antilepton of that generation and 0 for the other particles[4].

TABLE 2.1 Leptons (see Ref.1)

Leptons				Antileptons			
Q/|e|=-1	e^-	μ^-	τ^-	Q/|e|=+1	e^+	μ^-	τ^-
Q/|e|=0	ν_e	ν_μ	ν_τ	Q/|e|=0	$\overline{\nu_e}$	$\overline{\nu_\mu}$	$\overline{\nu_\tau}$

$m_e = 0.511 MeV/c^2$

$m_\mu = 105.6 MeV/c^2$

$m_\tau = 1870 MeV / c^2$

2.2 Quarks

Quarks, which are the indivisible building blocks of the hadrons, are spin 1/2 fermions and carry fractional charges (2/3|e| , -1/3|e|). They exist in six flavours : up(u), down(d), strange(s), charmed(c), bottom(b) and top(t). One can distinguish between them by the assignment of internal quantum numbers. Moreover, quarks can be arranged, just like leptons. in distinct doublets or generations[2,3,4] :

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$$

The upper member of each doublet (u, c, or t) has electric charge Q=2/3|e| while, the lower member carries charge Q=-1/3|e|. For each quark there are corresponding antiquarks which are denoted

$$\begin{pmatrix} \bar{d} \\ \bar{u} \end{pmatrix} \begin{pmatrix} \bar{s} \\ \bar{c} \end{pmatrix} \begin{pmatrix} \bar{b} \\ \bar{t} \end{pmatrix}$$

with charge's +1/3|e| for the upper members and -2/3|e| for the lower members. Quarks and antiquarks interact via strong, electromagnetic and weak interactions and the carriers or mediators of these distinct interactions are the spin 1 bosons. The u-and d- quarks are grouped in an isospin doublet (I=1/2). The s-quark is assigned an internal quantum number called strangeness (S=-1). The c-,b-and t-quark are assigned internal quantum numbers +1,-1and +1 which are known as charm, bottom and top respectively. A summary of the quarks' charges, masses and their internal quantum numbers is given in table 2.2.

TABLE 2.2 **Quarks (see Ref.1)**

Quarks	
$Q/\lvert e\rvert=+2/3$	u, c, t
$Q/\lvert e\rvert=-1/3$	d, s, b
u(I=1/2)	$m_u \approx 350MeV/c^2$
d(I=1/2)	$m_d \approx m_u \approx 350MeV/c^2$
s(S=-1)	$m_s \approx 550MeV/c^2$
c(C=+1)	$m_c \approx 1800MeV/c^2$
b(B=-1)	$m_b \approx 4500MeV/c^2$
t(T=+1)	$m_t \approx 174GeV/c^2$

Hadrons are regarded as composites of quarks. Hadrons consist of two classes:the lowest lying half integral -spin baryons are composed of three valance quarks$(qqq)_{L=0}$ while the spin 0 and1 mesons are composites of a valance quark- antiquark pairs $(q\bar{q})_{L=0}$. The above simple picture is capable of explaining the internal quantum numbers and the mass spectra of hadrons. All baryons carry a quantum number called baryon number. The baryon number is 1 for baryons, -1 for antibaryons and zero for others[2].

We are going to concentrate in the following sections and chapters on weak interactions and specifically on one type of weak decays the so-called non- leptonic decay of hyperons.

2.3 The Weak Interactions

The development of weak interactions starts with the first observation of the slow process of β-decay. All hadrons and leptons experience weak interactions[4]. The weak coupling constant, which is characterized by the Fermi constant G_F, when made dimensionless, has the value[1]

$$G_F(\mathrm{GeV})^2 = 10^{-5}$$

Weak interactions among four fermions are described by V-A interaction where the matrix element has the following form[2]:

$$M = \frac{G_F}{\sqrt{2}}\{\overline{\psi_1}\gamma_\mu(1+\gamma_5)\psi_2\}\cdot\{\overline{\psi_3}\gamma_\mu(1+\gamma_5)\psi_4\} \tag{2.1}$$

Weak interactions are mediated by the intermediate vector bosons which are spin-1 particles. On this picture the matrix elements (2.1) hold when the momentum transfer squared q^2 in the process $1+2\rightarrow3+4$ is negligible compared to M_W^2 where M_W is the mass of the intermediate vector boson. There are three types of the intermediate vector bosons: the charged bosons W^+ and W^- and the neutral Z^0. Weak decays are characterized by long lifetimes, small cross sections and they do not conserve isospin I and strangeness S. Neutrinos, which are electrically neutral and massless particles, can only interact via weak interactions. Although neutrinos are frequently found among the products of weak decays, they are not the only products[3]. For example, a K^+ meson can have the following decays

$$K^+ \to \pi^0 \mu^+ \nu_e \qquad \text{semileptonic decay}$$
$$K^+ \to \pi^+ \pi^0 \qquad \text{non-leptonic decay}$$

Therefore weak decays can be classified according to whether they involve leptons only or leptons and hadrons or hadrons only in the initial and final states[3]. Thus there are three types of weak decays:

2.3a The purely leptonic processes

The purely leptonic processes involve only leptons and the most familiar example is the μ-meson decay[2]

$$\mu^- \to e^- \bar{\nu}_e \nu_\mu \qquad (2.2)$$

The Lagrangian for the decay process (2.2) is given by[2]

$$\begin{aligned} L_w &= -\frac{G_F}{\sqrt{2}} \{ i\bar{\nu}_\mu \gamma_\mu (1+\gamma_5)\mu \} . \{ i\bar{e}\gamma_\mu (1+\gamma_5)\nu_e \} \\ &= \frac{G_F}{\sqrt{2}} L_\mu^\mu \bar{L}_\mu^{(e)} \qquad (2.3) \end{aligned}$$

where L^{μ}_{μ} and $L^{(e)}_{\mu}$ are lepton currents associated with (μ meson , ν_{μ}) and (e^{-} , ν_e) respectively[2].

$$L^{(\mu)}_{\mu} = i\bar{\nu}_{\mu}\gamma_{\mu}(1+\gamma_5)\mu$$

$$L^{(e)}_{\mu} = i\bar{\nu}_{e}\gamma_{\mu}(1+\gamma_5)e \tag{2.4}$$

The γ_{μ} and $\gamma_5(\equiv\gamma_1\gamma_2\gamma_3\gamma_4)$ are the usual Dirac matrices. The appearance of γ_5 in the weak current accounts for the fact that parity is violated maximally by weak interactions[1,3]. We can write the lepton current L_{μ} as a sum[2] of $L^{(\mu)}_{\mu}$ and $L^{(e)}_{\mu}$

$$L_{\mu} = L^{(\mu)}_{\mu} + L^{(e)}_{\mu} \tag{2.5}$$

while

$$\bar{L}_{\mu} = L^{+}_{\mu}\left(1-2\delta_{\mu 4}\right) \tag{2.6}$$

Figure (2.1) shows the process (2.2) with its mediator W_{μ} ,the so called weak vector boson.

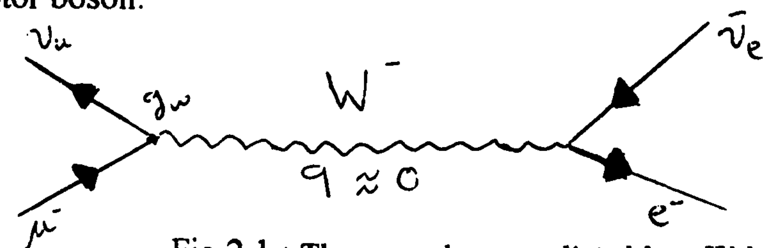


Fig.2.1 : **The muon decay mediated by a W-boson**

We can describe all leptonic weak processes analogous to electromagnetic interaction by[2]

$$L_w = g_w L_\mu W_\mu^- + \text{h.c.} \tag{2.7}$$

where h.c denotes the hermitian conjugate.

2.3b The semileptonic processes

The semi-leptonic processes contain leptons as well as hadrons in the initial and final state. The well-known example is the neutron β-decay[2,3]

$$n \rightarrow p e^- \bar{\nu}_e \tag{2.8}$$

In the semi-leptonic processes, the hypercharge is either conserved (Δ Y=0, ΔS=0) or changed by one unit ($\Delta|Y|=1, |\Delta S|=1$). Some examples of the hypercharge -conserving and the hypercharge-changing semi-leptonic decays are given below[2.5]

$$\Delta S = 0 \quad \begin{cases} n \rightarrow p + e^- + \bar{\nu}_e \\ \pi^+ \rightarrow e^+ + \nu_e, \mu^+ + \nu_\mu \\ \pi^- \rightarrow e^- + \bar{\nu}_e, \mu^- + \nu_\mu \\ \Sigma^- \rightarrow \Lambda^0 + e^- + \bar{\nu}_e \end{cases} \tag{2.9a}$$

$$|\Delta S| = 1 \quad \begin{cases} \Sigma^- \rightarrow n + e^- + \bar{\nu}_e \\ \Sigma^0 \rightarrow p + e^- + \bar{\nu}_e \\ K^+ \rightarrow \pi^0 + e^+ + \nu_e \\ K^- \rightarrow \pi^0 + e^- + \nu_e \end{cases} \tag{2.9b}$$

One can see from the above semi-leptonic decays that the hadronic charges change by one unit $|\Delta Q|=1$. The Gell-Mann-Nishijima relation[2]

$$Q=I_3+Y/2 \qquad (2.10)$$

implies that for hadrons with $|\Delta Q|=1$, either $\Delta I=\pm 1, \Delta Y=0$ or $\Delta I=\pm 1/2, \Delta Y=\pm 1$. Now hadrons are made up of quarks,three of which are of our interest u, d, s. Table2.3 gives the internal quantum numbers of the u, d, s quarks.

TABLE 2. 3 **The internal quantum numbers of u, d and s quarks**

	u-quark	d-quark	s-quark
I	1/2	1/2	0
I_3	1/2	-1/2	0
S	0	0	-1
B	1/3	1/3	1/3
Y=B+S	1/3	1/3	-2/3
Q	2/3	-1/3	-1/3

The interaction responsible for the fundamental processes like[2]

$$d \rightarrow u + e^- + \bar{\nu}_e$$
$$s \rightarrow u + e^- + \bar{\nu}_e \qquad (2.11)$$

would be

$$L_W^{h.c} = \frac{G_F}{\sqrt{2}} J_\mu^h \bar{L}_\mu^e$$

or

$$L_W = g_W J_\mu^h W_\mu^- + h.c \qquad (2.12)$$

where[2]

$$J_\mu^h = i\bar{u}\gamma_\mu (1 + \gamma_5) d' \qquad (2.13)$$

and

$$d' = \cos\theta_c d + \sin\theta_c s. \qquad s' = -\sin\theta_c d + \cos\theta_c s \qquad (2.14)$$

where θ_c denote the Cabibbo angle which is introduced to account for the suppression of $\Delta S=1$ transitions by a factor of 1/(16) as compared to the transitions with $\Delta S=0$ so that $\sin\theta c \approx 1/4$ .The d-and s-quarks, which are eigenstates of the mass Hamiltonian, are not identical with eigenstates of weak interaction Hamiltonian[1] which are d',s' given in Eq.(2.14). Figure(2.2) shows the quark level processes for neutron β-decay[2], $\Lambda - \beta -$ decay and $K^0 \rightarrow \pi^+ + e^- + \bar{\nu}_e$

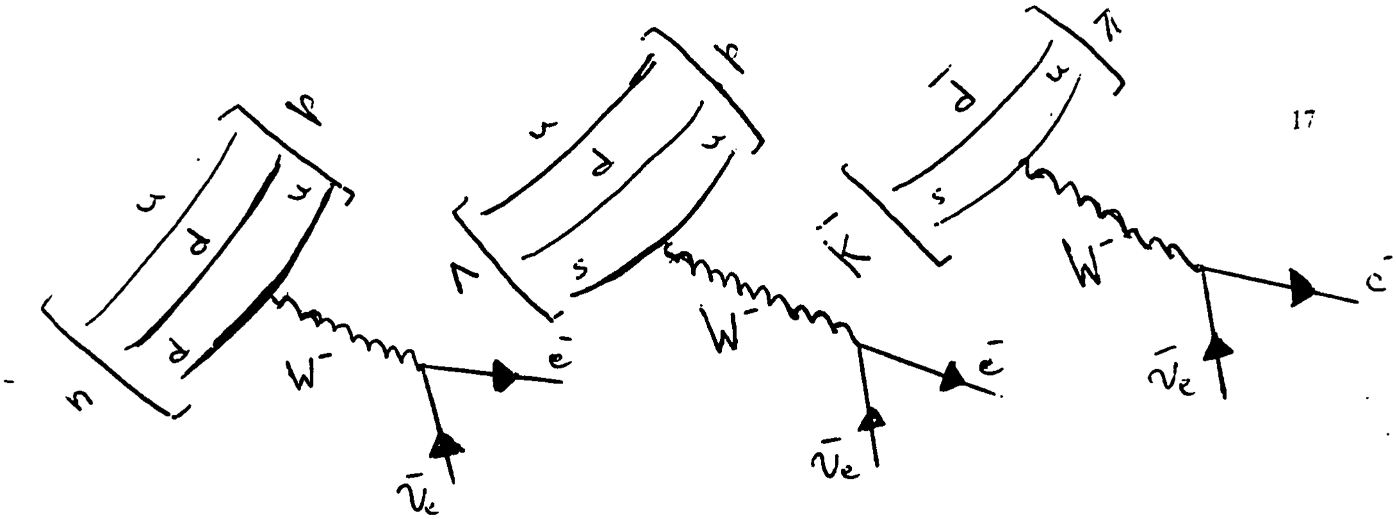


Fig.2.2:**Quark level processes for nutron β-decay, Λ-βdecay and** $\overline{K}^0 \to \pi^+ e^- \bar{\nu}_e$

2.3c The Non - Leptonic Hyperon Decays

Non-leptonic processes or weak hadronic processes do not involve any leptons in the initial and final state. The following decays are have been observed for non-leptonic processes[2,5]

$$\begin{aligned} &\Lambda \to p\pi^- &&\left(\Lambda^0_-\right) \\ &\Lambda \to p\pi^0 &&\left(\Lambda^0_0\right) \end{aligned} \tag{2.15a}$$

$$\begin{aligned} &\Sigma^- \to n\pi^- &&\left(\Sigma^-_-\right) \\ &\Sigma^+ \to p\pi^0 &&\left(\Sigma^+_0\right) \\ &\Sigma^+ \to n\pi^+ &&\left(\Sigma^+_+\right) \end{aligned} \tag{2.15 b}$$

$$\begin{aligned} &\Xi^- \to \Lambda\pi^- &&\left(\Xi^-_-\right) \\ &\Xi^0 \to \Lambda\pi^0 &&\left(\Xi^0_0\right) \end{aligned} \tag{2.15c}$$

We devote the rest of the discussion on the decays (2.15) the so called non- leptonic decays of hyperons. These non- leptonic decays of strange particles are characterized by the selection rules $|\Delta S|=1$ and $\Delta I=1/2$. This is expected, if one replaces the quark s (S=-1, I=0) with a non strange quark d, u (S=0 , I=1/2)[1] .In the decay process of a hyperon in its rest frame, the conservation of angular momentum implies that[2]

$$J_{initial} = 1/2 = J_{final} = l + s$$

where l is the relative orbital angular momentum of the pion and the baryon in the final state. Since the spin s=1/2, we can have either l=0 or l=1. Thus, one expects to have a combination of s-and p-waves. The parity of the final state is $(-1)^{l+1}$ since the pion is pesudoscalar (having odd intrinsic parity). Thus the s-wave (l=0) implies that $P_f = (-1)^0(1) = -1$(odd), while the p - wave (l=1) implies that $P_f = (-1)^1(-1) = 1$ (even). Thus, the s- wave (l=0) arises from the parity violating part of H_W while the p-wave (l=1) arise from the parity conserving part[2,6].

The non- leptonic Lagrangian, which is responsible for the weak hadronic processes, has the following form[2]

$$L_w^{(h)} = \frac{G_F}{\sqrt{2}} J_\mu \overline{J_\mu} + h.c \tag{2.16}$$

where $J_\mu = i\bar{u}\gamma_\mu(1+\gamma_5)d'$. The $|\Delta S=1|$ component of Eq.(2.16) gives the Hamiltonian (H_W) responsible for the non-leptonic decay of hyperons[2].

$$H_W = \frac{G_F}{\sqrt{2}} \sin\theta_c \cos\theta_c \left\{\bar{s}\gamma_\mu(1+\gamma_5)u\right\} \cdot \left\{\bar{u}\gamma_\mu(1+\gamma_5)d\right\} \tag{2.17}$$

where $G_F = 11.6\times10^{-6}GeV^{-2}$, is the Fermi coupling constant and $\sin\theta_c = 0.221$.

Now, let us consider the weak hadronic decay of hyperon B

$$B(p) \to B(p') + \pi(k)$$

where B and B′ are the members of baryon octet and denote the initial and final state respectively. The reduced T-matrix is given by[2,5]

$$\begin{aligned} T &= -\langle B'\pi | H_w^h(0) | B\rangle \\ &= \frac{1}{(2\pi)^{9/2}} \sqrt{\frac{mm'}{2k_0 p_0 p_0'}} \quad \bar{u}(p')[A + B\gamma_5]u(p) \end{aligned} \tag{2.18}$$

where A and B, in the rest frame of B, are constant amplitudes that represent the parity violating part (s-wave) and the parity conserving part (p-wave) respectively. In this rest frame[2,5]

$$u(\bar{p}) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}\chi, \quad u(\bar{p}') = \frac{1}{\sqrt{2m'(p_0' + m')}} \begin{pmatrix} m' + \bar{p}_0' \\ \bar{\sigma}.\bar{p}' \end{pmatrix} \tag{2.19}$$

where χ is a constant 2-component Pauli spinor. The T-matrix can be rewritten in the following form[2]

$$T = \chi^{+} M \chi \tag{2.20}$$

where the 2×2 matrix M is given by[2,5]

$$M = \frac{1}{(2\pi)^{9/2}} \frac{1}{\sqrt{2k_0}} \left[a_s + a_p \vec{\sigma}.\vec{n} \right] \tag{2.21}$$

where

$$a_s = \sqrt{\frac{p_0' + m'}{2p_0'}} A, \quad a_p = \frac{k}{\sqrt{2p_0'(p_0' + m')}} B \ ,$$

$$k = \frac{1}{2m} \sqrt{\left[m^2 - (m' - m_\pi)^2 \right] \left[m^2 - (m' + m_\pi)^2 \right]} \ , \tag{2.22}$$

and

$$p_0' = \frac{m^2 + m'^2 + m_\pi^2}{2m} \tag{2.23}$$

The angular distribution of the decay baryon B' relative to the polarization or spin direction of B is given by[5]

$$I(\theta) = \mathrm{const}(1 + \alpha S \cos\theta) \tag{2.24}$$

where θ is the angle between the hyperon spin s and the decayed baryon's momentum direction. α ,which produces the antisotropy of the angular distribution,thereby imply that the interaction as whole violate parity conservation[1,2,5], is given by

$$\alpha = \frac{2\,\mathrm{Re}\left(a_s^* a_p\right)}{|a_s|^2 + |a_p|^2}$$

2.4 ΔI=1/2 Rule

The ΔI=1/2 rule states that in non-leptonic weak decays the total isos2pin changes by 1/2. This rule is an experimentally observed rule and seems to govern all weak decays involving hyperons. To clearify this point, let us consider the decays

$$\Lambda^0 \to p + \pi^- \qquad I_i = 0,\ I_f = 1/2, 3/2,$$
$$\Lambda^0 \to n + \pi^0 \qquad I_i = 0,\ I_f = 1/2, 3/2,$$

From a table of Clebsch - Gordan coefficients. we can express the decay rates of the above two processes in terms of ΔI=1/2 and ΔI=3/2 amplitudes. If the ΔI=1/2 rule holds, the ΔI=3/2 amplitudes are all zero. In that case, the branching ratio into $\pi^- p$ mode is 2/3. Experimentally the branching ratio of that decay mode is 0.653±0.013,which is in a good agreement with the ΔI=1/2 rule prediction[7].

The u- and d- quarks are an isospin doublet, I=1/2, while s is isospin singlet, I=0. Thus the first curly bracket of Eq (1.17) has I=1/2 while the second curly bracket has I=0,1. Thus the interaction given in Eq(1.17) contains both $\Delta I=1/2$, $\Delta I=3/2$. Empirically the $\Delta I=1/2$ amplitude is enhanced and the $\Delta I=3/2$ amplitude is suppressed[4]. There is no satisfactory understanding of $\Delta I=1/2$ rule. Let us now consider the $|\Delta I|=1/2$ enhancement from the point of view of flavor SU(3). We know that u, d and s belong to 3 representation of ordinary flavor SU(3) while $\bar{u}, \bar{d}$ and $\bar{s}$ belong to the $\bar{3}$ representation [2]. Now

$$3 \otimes \bar{3} = 8 \oplus 1$$

Thus J_μ belongs to the octet representation of ordinary flavour SU(3). Now, when one takes the tensor product of the two octets in SU(3), one obtains the following irreducible representations[2,4]

$$8 \otimes 8 = 27 \oplus 10 \oplus \overline{10} \oplus 8_S \oplus 8_A \oplus 1$$

The $27, 8_S$ and 1 are symmetric while 10, $\overline{10}$ and 8_A are antisymmetric. We are concerned with the portion of H_W that can provide weak transitions with $|\Delta S|=1$. Thus, the 27 and 8_S are the only relevant components. The 8_S contains $\Delta I=1/2$, whereas the 27 contains $\Delta I=1/2$ as well as $\Delta I=3/2$. So, the $|\Delta I|=1/2$ rule implies that the 8_S is enhanced or the 27 is suppressed. This is known as the octet dominance[4]. The $\Delta I=1/2$ rule is an approximate rule and thus $\Delta I=3/2$ shoud be expected to occur with a smaller amplitude.

Now, using the Wigner-Eckhart theorem, one can derive useful relations between various hyperone non-leptonic decay amplitudes .Referring to the Clebsh-Gordan coefficient $\langle I_1, m_1, I_2, m_2 | I, m_1 + m_2 \rangle$, the $\Delta I=1/2$ rule predicts for s-wave the following relation[4]

$$A(\Lambda_-^0)/A(\Lambda_0^0) = \langle 1,-1,1/2,1/2 | 1/2,-1/2\rangle / \langle 1,0,1/2,-1/2 | 1/2,-1/2\rangle$$
$$= -\sqrt{2}$$

so that[2,4,5]

$$A\left(\Lambda_-^0\right) = -\sqrt{2}A\left(\Lambda_0^0\right)$$

Similarly

$$A\left(\Sigma_+^+\right) - A\left(\Sigma_-^-\right) = +\sqrt{2}A\left(\Sigma_0^+\right)$$
$$A\left(\Xi^0\right) = -\frac{1}{\sqrt{2}}A\left(\Xi^-\right) \qquad (2.25a)$$

In addition to these relations, which follow from $|\Delta I| = 1/2$ rule, we may obtain one additional relationship, the so called Lee- Sugawara (L-S) relation for the s-wave amplitudes which is a consequence of CP invariance and the assumption that p.v part of H_w^h transforms like λ_6 [5], :

$$2A(\Xi_-^-) + A(\Lambda_-^0) = \sqrt{3}\, A(\Sigma_0^+) \qquad (2.25b)$$

In the same way, one can obtain same relations for the p-wave amplitudes

$$B\left(\Lambda_-^0\right) + \sqrt{2}B\left(\Lambda_0^0\right) = 0$$
$$B\left(\Xi_-^-\right) + \sqrt{2}B\left(\Xi_0^0\right) = 0 \qquad (2.26a)$$
$$B\left(\Sigma_-^-\right) + \sqrt{2}B\left(\Sigma_0^+\right) - B\left(\Sigma_+^+\right) = 0$$

The L-S sume rule for the p-wave amplitudes[5] is

$$B(\Lambda^0_-) + \sqrt{2}B(\Xi^-_-) = \sqrt{3}B(\Sigma^+_0) \tag{2.26b}$$

2.5 Application of Current Algebra to Hadronic Decays

The universal current-current theory of weak hamiltonian describes both the leptonic and semi-leptonic processes. However, applying the theory to the non-leptonic decays,even when octet dominance is assumed, there are complications due to strong interactions when the matrix elements of the hamiltonian are taken between relevant hadron states. One approach which has been fairly successful is that of current algebra and partial conservation of axial vector current(PCAC). This approach, we briefly review.

The octet of vector and axial vector currents are given by[2,5]

$$\begin{aligned} V_{i\lambda} &= i\bar{\psi}\frac{\lambda_i}{2}\gamma_\lambda\psi \\ A_{i\lambda} &= i\bar{\psi}\frac{\lambda_i}{2}\gamma_\lambda\gamma_5\psi \end{aligned} \tag{2.27}$$

where $\psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$ and $\bar{\psi} = \begin{pmatrix} \bar{u} & \bar{d} & \bar{s} \end{pmatrix}$ and λ_i are 3×3 Gell – Mann matrices

We may define the generators of SU(3)×SU(3) group of vector and axial vector charges by[2,5]

$$F_i = -i\int V_{i4}(x,t)d^3x \qquad (i=1,\ldots\ldots\ldots,8) \qquad (2.28)$$

$$F_i^5 = -i\int A_{i4}(x,t)d^3x \qquad (i=1,\ldots\ldots\ldots,8) \qquad (2.29)$$

These generators satisfy the folowing equal time commutation relations[5]

$$\left[F_i, F_j\right] = if_{ijk}F_K \qquad (2.30a)$$

$$\left[F_i, F_j^5\right] = if_{ijk}F_K^5 \qquad (2.30b)$$

$$\left[F_i^5, F_j^5\right] = if_{ijk}F_k \qquad (2.30c)$$

$$\left[F_i, V_{j\mu}\right] = if_{ijk}V_{k\mu}, \quad \left[F_i, A_{j\mu}\right] = if_{ijk}A_{k\mu} \qquad (2.31a)$$

$$\left[F_i^5, V_{j\mu}\right] = if_{ijk}A_{k\mu}, \quad \left[F_i^5, A_{j\mu}\right] = if_{ijk}V_{j\mu} \qquad (2.31b)$$

where f_{ijk} are sructure constants of the SU(3) group. Now. from the above commutation relations. we obtain[2,5]

$$\left[F_i^5, H_w^h\right] = \left[F_i, H_w^h\right], \quad \left[F_i^5, H_w^{p.c,p.v}\right] = \left[F_i, H_w^{p.v,p.c}\right] \qquad (2.32)$$

In the calculation of the decay amplitudes, for a process of the type

$$B_r(p) \rightarrow B_s(p') + \pi(k)$$

we encounter matrix elements of the type

$$\left\langle B_s(p')\pi_i(k)\middle|H_w\middle|B_r(p)\right\rangle$$

To deal with such matrix elements, we shall make use of the following basic relation of spontaneously broken SU(2) symmetry generated by $F_i^5 (i = 1,2,3)$

$$F_i^5|0\rangle = -\frac{i}{2}F_\pi |\pi_i(k=0)\rangle \tag{2.33}$$

where F_π is the decay constant defined by[2,5]

$$\langle 0|A_{i\mu}|\pi_i(k)\rangle = iF_\pi k_\mu \tag{2.34}$$

Here i= $\frac{1 \pm i2}{\sqrt{2}}$,3 corresponding to $\pi^\pm$ or π^0. Then applying the relation (2.33) and its hermitian conjugate to the following matrix elements

$$\begin{aligned}&\langle B_s(p')|[F_i^5, H_w]|B_r(p)\rangle \\ &\quad = \langle B_s(p')|F_i^5 H_w - H_w F_i^5|B_r(p)\rangle\end{aligned}$$

we obtain

$$\begin{aligned}&\langle B_s(p')|[F_i^5, H_w]|B_r(p)\rangle \\ &\quad = \frac{i}{2}F_\pi\left[\langle B_s(p')\pi_i(0)|H_w|B_r(p)\rangle + \langle B_s(p')|H_w|\pi_i(0)B_r(p)\rangle\right] \\ &\quad = iF_\pi\langle B_s(p')\pi_i(0)|H_w|B_r(p)\rangle \end{aligned} \tag{2.35}$$

which holds in the soft pion limit where $k_\mu=(p-p')_\mu\to 0$. The Eq.(2.35) gives in this limit[2]

$$\lim_{k\to 0}\langle B_s(p')\pi_i(k)|H_w|B_r(p)\rangle=-\frac{i}{F_\pi}\left\langle B_s(p')\left|\left[F_i^5,H_w\right]\right|B_r(p)\right\rangle \tag{2.36}$$

We can not apply the above technique directly to p-wave or p.c amplitudes since those amplitudes are singular at k=0. However. the above mentioned singularity arises due to baryon poles (Born terms) shown which are shown in Fig.2.3. By subtracting out the Born terms. we can apply the above technique to the residual amplitude[2,5,6]. Thus, we have for the s-waves or p.v amplitudes[2], on using Eq.(2.32),

$$\begin{aligned}\lim_{k\to 0}\left\langle B_s(p')\pi_i(k)\left|H_w^{p.v}\right|B_r(p)\right\rangle&=-\frac{i}{F\pi}\left\langle B_s(p')\left|\left[F_i^5,H_w^{p.v}\right]\right|B_r(p)\right\rangle\\&=-\frac{i}{F_\pi}\left\langle B_s(p')\left|\left[F_i,H_w^{p.c}\right]\right|B_r(p)\right\rangle\end{aligned} \tag{2.37}$$

By using the relations of the type[5]

$$\begin{aligned}&\langle p|F_+=\langle n|,\quad F_+|\Lambda\rangle=0,\quad \langle n|F_3=-\frac{1}{2}\langle n|\\&F_+\left|\Xi^-\right\rangle=-\left|\Xi^0\right\rangle,\quad F_3\left|\Xi^0\right\rangle=\frac{1}{2}\left|\Xi^0\right\rangle\\&F_+\left|\Sigma^-\right\rangle=\sqrt{2}\left|\Sigma^0\right\rangle,\quad F_-\left|\Sigma^+\right\rangle=-\sqrt{2}\left|\Sigma^0\right\rangle\end{aligned}$$

Eq.(2.37) gives the following expressions for individual s-wave amplitudes[2,5]:

$$
\begin{aligned}
A\left(\Lambda_-^0\right) &= -\frac{1}{\sqrt{2}F_\pi} a_{\Lambda n} = -\sqrt{2} A\left(\Lambda_0^0\right) \\
A\left(\Xi_-^-\right) &= -\frac{1}{\sqrt{2}F_\pi} a_{\Xi^0 \Lambda} = -\sqrt{2} A\left(\Xi_0^0\right) \\
A\left(\Sigma_0^+\right) &= \frac{1}{2F_\pi} a_{\Sigma^+ p} \\
A\left(\Sigma_+^+\right) &= -\frac{1}{\sqrt{2}F_\pi}\left(a_{\Sigma^+ p} + \sqrt{2} a_{\Sigma^0 n}\right) \\
A\left(\Sigma_-^-\right) &= \frac{1}{F_\pi} a_{\Sigma^0 n}
\end{aligned}
\tag{2.38}
$$

where $a_{\Lambda n} = \left\langle n \middle| H_w^{p.c} \middle| \Lambda \right\rangle$ etc . As remarked earlier, for a p-wave amplitude we can apply the result of Eq(2.35) to the residual amplitude, the one from which Born term has been subtracted out and thus we obtain[2]

$$
\begin{aligned}
\lim_{k\to 0}&\left[\left\langle B_s(p')\pi_i(k) \middle| H_w^{p.v} \middle| B_r(p)\right\rangle - \left\langle B_s(p')\pi_i(k) \middle| H_w^{p.c} \middle| B_r(p)\right\rangle_{\text{Born}}\right] \\
&= -\frac{i}{F_\pi}\left\langle B_s(p') \middle| \left[F_i^5, H_w^{p.c}\right] \middle| B_r(p)\right\rangle \\
&= -\frac{i}{F_\pi}\left\langle B_s(p') \middle| \left[F_i, H_w^{p.v}\right] \middle| B_r(p)\right\rangle
\end{aligned}
\tag{2.39}
$$

It may be noted that for the s-wave the Born terms vanish as a consequence of CPand SU(3) invariance which gives $\left\langle B_s \middle| H_w^{p.v} \middle| B_r \right\rangle \approx 0$ and for the same reason the term on the right hand side of Eq (1.39) also vanishes[2,5]. Thus, the p-waves in the soft pion limit are given by Born terms only which involve the weak matrix elements $\left\langle B_s(p') \middle| H_w^{p.c} \middle| B_r(p) \right\rangle$, the same ones which appear in the s-wave amplitudes as given in Eq.(2.36).

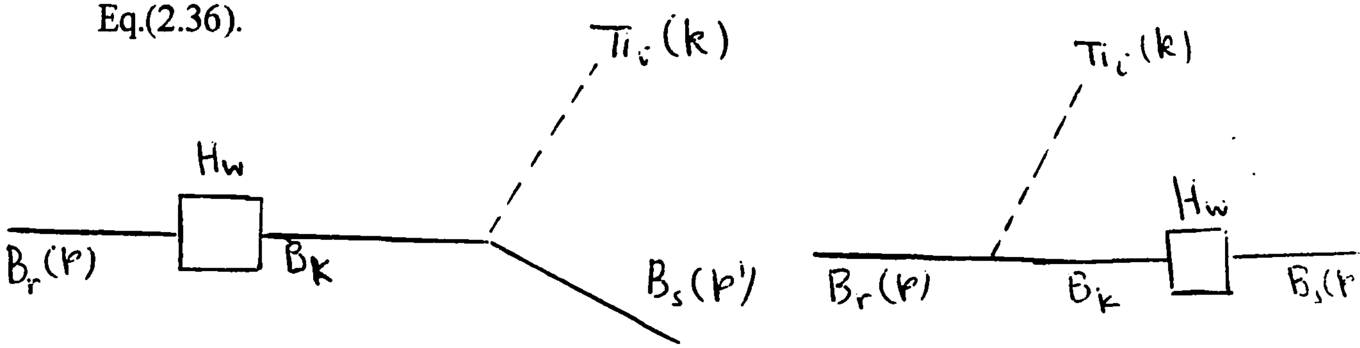


Fig.2.3 : **Pole diagram in hyperon decays**

In the flavor SU(3) limit, these matrix elements can be parametrized in terms of two reduced matrix elements[2] F and D

$$\left\langle B_s \middle| H_w^{p.c} \middle| B_r \right\rangle = 2\sqrt{2}\,\bar{u}\left(i f_{6ij} F + d_{6ij} D\right) u \tag{2.40}$$

From the above discussion, one can obtain the following expressions for the s- and p-waves:

s-waves

$$\begin{aligned} A\left(\Sigma_-^-\right) &= \frac{1}{F_\pi}(-D+F) \\ A\left(\Sigma_+^+\right) &= 0 \\ A\left(\Sigma_0^+\right) &= \frac{1}{\sqrt{2}F_\pi}(D-F) \\ A\left(\Xi_-^-\right) &= \frac{1}{\sqrt{6}F_\pi}(D+3F) \end{aligned} \tag{2.41}$$

p-waves

$$
\begin{aligned}
B\left(\Sigma_-^-\right) &= 2g(N+\Sigma)\left[\frac{f(F-D)}{(\Sigma-N)2\Sigma}-\frac{d(3F+D)}{3(\Lambda-N)(\Sigma+\Lambda)}\right] \\
B\left(\Sigma_+^+\right) &= 2g(N+\Sigma)\left[\frac{(f+d)(F-D)}{(\Sigma-N)2N}-\frac{f(F-D)}{(\Sigma-N)2\Sigma}-\frac{d(3F+D)}{3(\Lambda-N)(\Sigma+\Lambda)}\right] \\
B\left(\Sigma_0^+\right) &= \sqrt{2}g(N+\Sigma)\left[\frac{(f+d)(F-D)}{(\Sigma-N)2N}-\frac{2f(F-D)}{(\Sigma-N)2\Sigma}\right] \\
B\left(\Lambda_-^0\right) &= \sqrt{\frac{2}{3}}g(N+\Lambda)\left[\frac{(f+d)(3F+D)}{(\Lambda-N)2N}-\frac{2d(F-D)}{(\Sigma-N)(\Sigma+\Lambda)}\right] \\
B\left(\Xi_-^-\right) &= \sqrt{\frac{2}{3}}g(\Lambda+\Xi)\left[-\frac{2d(F+D)}{(\Xi-\Sigma)(\Sigma+\Lambda)}+\frac{(d-f)(3F-D)}{(\Xi-\Lambda)2\Xi}\right]
\end{aligned}
\tag{2.42}
$$

where $F_\pi \approx 93$MeV, g is the pion-nucleon coupling constant $g^2/4\pi = 14.6$, f and d are the corresponding reduced matrix elements baryon - baryon-pseudoscalar meson coupling constants (f+d=1). Table2.4 gives the experimental values of hyperon decay amplitudes[7].

TABLE 2.4 **Observed hyperon decay amplitudes(see Ref.7)**

	s wave $10^6 A$	p wave $10^6 B$
$\left(\Lambda^0_-\right)$	0.323±0.002	2.20±0.05
$\left(\Lambda^0_0\right)$	-0.237±0.003	-1.59±0.14
$\left(\Sigma^+_0\right)$	-0.326±0.011	2.67±0.15
$\left(\Sigma^+_+\right)$	0.014±0.003	4.22±0.01
$\left(\Sigma^-_-\right)$	0.427±0.002	-0.14±0.02
$\left(\Xi^-_-\right)$	-0.450±0.002	1.75±0.06
$\left(\Xi^0_0\right)$	0.344±0.006	-1.22±0.07

CHAPTER 3

Non-leptonic Hyperon Decays in Non-Relativistic Quark Model

3.1 Introduction: The Quark Model

According to the quark model, the low lying baryons are composed of three quarks $(qqq)_{L=0}$ bound together while the low lying mesons are made up of a quark and an antiquark $(q\bar{q})_{L=0}$ bound together[2,8]. The wave function for any composite particle can be written as[9]

$$\Psi = \Psi_{space}\Psi_{spin}\Psi_{flavor} \tag{3.1}$$

The space, spin and flavor wave functions are symmetric under the exchange of any two quarks but this violates the Pauli principle which requires a total wave function of fermions to be antisymmetric. One way out of this difficulty is to introduce an additional factor to the combined wave function making it as a whole antisymmetric. This factor involves a new quantum number which is known as color. The three colors are labled red(R), blue(B) and green(G). An important assumption in the quark model is that all hadrons are color singlets, they are colorless, this requires that[9]

$$(qqq)_{L=0} \rightarrow \epsilon_{abc} \left(q^a q^b q^c\right)_{L=0}$$

where a,b,c=1,2,3 are color indices. This shows antisymmetry in color indices. Now the quark wave function becomes

$$\Psi = \Psi_{space} \Psi_{spin} \Psi_{flavor} \Psi_{color} \qquad (3.2)$$

where Ψ_{color} is antisymmetric under the interchange of color so that the total wave function is also antisymmetric.

3.2 The Low Lying Mesons

For three flavours of quarks, q=u,d and s there are nine possible $q\bar{q}$ combinations. The only flavor SU(3) multiplets that are allowed are the SU(3) singlet and the SU(3) octet. Since the quark has spin 1/2 the intrinsic spin for mesons can be either S=0 or S=1 and the total angular momentum J of the composite meson is the vector sum of the spin S=0,1 and L=0 which gives J=0,1. The parity P of the meson is given by[2,8]

$$P = (-1)^{L+1} = (-1)(-1)^0 = -1 \qquad (3.3)$$

The charge conjugation parity C is given by

$$C=(-1)^{S+L}=(-1)^{0}(-1)^{S}=\begin{cases} S=0, C=+1 \\ S=1, C=-1 \end{cases} \tag{3.4}$$

The pseudoscalar meson states (forming an octet under flavor SU(3)) and the vector meson states(forming a nonet under flavor SU(3)) are given for $J^{PC}=0^{-+},1^{--}$ respectively . These together represent 35 states 8 $\times 1+9\times 3$, where 1 and 3 represent spin singlet and spin triplet states forming an irreducible 35 dimentional representation[10] of SU(6):

$$6\otimes\bar{6}=35\oplus 1 \tag{3.5}$$

For the pseudoscalar mesons the spin for the quark and the antiquark are antiparallel so that the spin wave function is singlet and is given by[2,8]

$$\chi_A^0=\frac{1}{\sqrt{2}}\left|\uparrow\downarrow-\downarrow\uparrow\right\rangle \tag{3.6}$$

For the vector mesons the quark and the antiquark are parallel so that the spin wave function is triplet and the spin triplet states are given by

$$\chi_S^{1,0,-1} = |\uparrow\uparrow\rangle, \frac{1}{\sqrt{2}}|\uparrow\downarrow+\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle \tag{3.7}$$

Let's give some examples that are related to our disscusion[2]

$$\begin{aligned}
\pi^+ &= \frac{1}{\sqrt{2}}\left(\bar{d}^{\uparrow}u^{\downarrow} - \bar{d}^{\downarrow}u^{\uparrow}\right) \\
\pi^- &= \frac{1}{2}\left(\bar{u}^{\uparrow}u^{\downarrow} - \bar{u}^{\downarrow}u^{\uparrow} - \bar{d}^{\uparrow}d^{\downarrow} + \bar{d}^{\downarrow}d^{\downarrow}\right) \\
K^{*0}(J_z = 0,1,-1) &= \frac{1}{\sqrt{2}}\left(\bar{s}^{\uparrow}d^{\downarrow} + \bar{s}^{\downarrow}d^{\uparrow}\right),\ \bar{s}^{\uparrow}d^{\downarrow},\ \bar{s}^{\downarrow}d^{\downarrow}
\end{aligned} \tag{3.8}$$

3.3 The Low Lyning Baryons

The low lyning baryons are made up of three quarks $(qqq)_{L=0}$, the parity $P = (-1)^0(1)^3=1$. For the three flavors u, d and s, there are 27 possible qqq combinations. The qqq SU(3) multiplets are singlets, octets and decuplets[8,10].

$$3\otimes3\otimes3 = 10\oplus8_S\oplus8_A\oplus1 \tag{3.9}$$

In the ground state, the baryon spin J is found by the addition of the three spins 1/2. If we apply the SU(2) group for the three spin1/2 states, one can obtain four symmetric states, two mixed symmetric states and two mixed antisymmetric states[10]. This can be shown as follows[10]

$$\frac{1}{2}\times\frac{1}{2}=1_S+0_A\text{, so that}$$

$$\left(\frac{1}{2}\times\frac{1}{2}\right)\times\frac{1}{2}=\left(1_S\times\frac{1}{2}\right)+\left(0_A\times\frac{1}{2}\right)$$

$$=\frac{3}{2}(S)+\frac{1}{2}(M_S)+\frac{1}{2}(M_A) \tag{3.10}$$

and in terms of the multiplicities

$$2\otimes 2\otimes 2=(3\oplus 1)\otimes 2=4_S\oplus 2_{M_S}\oplus 2_{M_A}$$

One can construct the SU(6) represntations of qqq that arise when the u,d and s quarks with spin 1/2 form six dimentional fundamental representation$(u\uparrow d\uparrow s\uparrow, u\downarrow d\downarrow s\downarrow)$. This is done by combining the 3 dimentional SU(3) representation (u,d and s) with SU(2) $(\uparrow\downarrow)$[2,10]. Thus the irreducible representations of SU(6) in this case are[10]

$$6\otimes 6\otimes 6=56_S\oplus 70_{M_S}\oplus 70_{M_A}\oplus 20_A \tag{3.11}$$

If we combine the SU(3) decomposition with the SU(2) spin decomposition[11]

$$\left(10_S\oplus 8_{M_S}\oplus 8_{M_A}\oplus 1_A\right)\quad ,\left(4_S\oplus 2_{M_S}\oplus 2_{M_A}\right) \tag{3.12}$$

we obtain the following categories[10]

$$\begin{aligned}&S:\ 56=(10\otimes 4)\oplus(8\otimes 2)\\&M_S,M_A:\ 70=(10\otimes 2)\oplus(8\otimes 4)\oplus(8\otimes 2)\oplus(1\otimes 2)\\&A:\ 20=(8\otimes 2)\oplus(1\otimes 4)\end{aligned} \tag{3.13}$$

The lowest lying states belong to the 56 representation of SU(6), which is completely symmetric.

3.4 The Non-Relativistic Quark Model for Non-leptonic Hyperon Decays

In the non-relativistic quark model, the internal dynamics of the constituent particles is treated in the non-relativistic limit[8]. This non-relativistic approach was first applied to parity conserving (p.c) hyperon $\Delta S=1$ weak transitions with encourging results[11,12]. By this approach one could recover not only $\Delta I=1/2$ rule but also the right order of magnitude of the scale required to reproduce the s- and p-wave fits of non-leptonic hyperon decays. This model involves the W- exchange transitions depicted in Figure (3.1).

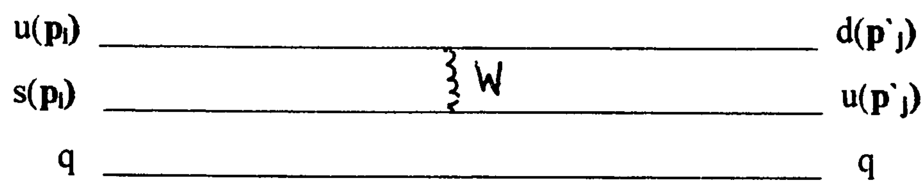


Figure 3.1: **Two-quark interaction within the baryon.**

The q (u,d or s) is spectator quark in baryon qqq structure. The matrix element for such a process described in Fig.(3.1) is of the form[11]

$$M\left(m_w^2 \gg q^2\right) \approx \frac{1}{2}\frac{g_w^2}{m_w^2}\sin\theta_c\cos\theta_c\left\{\bar{u}(p'_i)\gamma_\mu(1+\gamma_5)\alpha_i^- u(p_i)\right.$$

$$\left.\times\bar{u}(p'_j)\gamma_\mu(1+\gamma_5)\beta_j^+ u(p_j)+i\leftrightarrow j\right\} \qquad (3.14)$$

where u's are Dirac spinors. α_i^- and β_j^+ are operators which transform a u-quark state into a d-quark state and a s-quark state into a u-quark state respectively. We perform a non-relativistic expansion of the above matrix element for parity conserving part of the weak hamiltonian Eq(1.17). So, only the A.A-V^0V^0 terms will contribute in this limit.

$$\left(\bar{u}_i\gamma_\mu u_i\right)\left(\bar{u}_j\gamma_\mu u_j\right)+\left(\bar{u}_i\gamma_\mu\gamma_5 u_i\right)\left(\bar{u}_j\gamma_\mu\gamma_5 u_j\right) \tag{3.15}$$

The non- relativistic treatment of the above terms gives the following

$$\bar{u}\gamma u \approx -\frac{i}{2m}\left[(\bar{p}+\bar{p}')+i\bar{\sigma}\times(\bar{p}'-\bar{p})\right] \tag{3.16}$$

$$\bar{u}\gamma_4 u \approx \left[1+\frac{(\bar{p}+\bar{p}')^2}{8m^2}+\frac{i}{4m^2}\bar{\sigma}.(\bar{p}'\times\bar{p})\right] \tag{3.17}$$

$$\bar{u}\gamma\gamma_5 u \approx i\left[\bar{\sigma}+\frac{\left(\bar{p}^2+\bar{p}'^2\right)}{8m^2}\bar{\sigma}+\frac{(\bar{\sigma}.\bar{p}')\bar{\sigma}(\bar{\sigma}.\bar{p})}{4m^2}\right] \tag{3.18}$$

$$\bar{u}\gamma_4\gamma_5 u \approx -\frac{1}{2m}\left[\bar{\sigma}.(\bar{p}'+\bar{p})\right] \tag{3.19}$$

We have in this limit the simple operator for the parity conserving matrix element[1,2]

$$M^{p.c} \sim \frac{1}{\sqrt{2}} G_F \cos\theta_c \sin\theta_c \sum_{i\rangle j} \left(\alpha_i^- \beta_j^+ + \beta_i^+ \alpha_j^-\right)\left(1 - \bar{\sigma}.\bar{\sigma} \right) \tag{2.20}$$

where $\frac{G_F}{\sqrt{2}} = \frac{g_w^2}{m_w^2}$ and $\bar{s}_i = \frac{1}{2}\bar{\sigma}_i$. Let Ψ be the wave function in momentum space for any of octet baryons $p, n, \Lambda, \Sigma^{\pm}, \Sigma^0, \Xi^0, \Xi^-$. Then the flavor, spin matrix elements and space (the color matrix element is just equal to 1) are given by

$$(2\pi)^3 \left\langle B_s \middle| H_w^{p.c}(0) \middle| B_r \right\rangle = \frac{G_F}{\sqrt{2}} \sin\theta_c \cos\theta_c \left\langle B_s \middle| \sum_{i\rangle j} \left(\alpha_i^- \beta_j^+ + \beta_i^+ \alpha_j^-\right)\left(1 - \bar{\sigma}.\bar{\sigma} \right) \middle| B_r \right\rangle d' \tag{3.21}$$

where d\` is the spatial integral[13]

$$\begin{aligned} d' = (2\pi)^3 \int \psi^*\left(\bar{p}_1', \bar{p}_2', \bar{p}_3'\right) \psi\left(p_1, p_2, p_3\right) \delta\left(\bar{p}_3 - \overline{p_3'}\right) \\ \prod_i d\bar{p}_i \, d\bar{p}_i' \delta\left(\sum_j \bar{p}_j\right) \delta\left(\sum_k \bar{p}_k'\right) \end{aligned} \tag{3.22}$$

By taking the fourier transform in terms of the spatial wave function in configuration space[13]:

$$\psi\left([\bar{p}_i]\right) = \frac{1}{(2\pi)^3} \int \psi\left([\bar{r}_i]\right) \prod_j d\bar{r}_j \delta\left(\frac{1}{3} \sum_k \bar{r}_k\right) e^{i \sum_l \bar{p}_l . \bar{r}_l} \tag{3.23}$$

one gets[2,13]

$$d' = \left\langle \psi \middle| \delta^3(\bar{r}) \middle| \psi \right\rangle = |\Psi(0)|^2 \tag{3.24}$$

where $\bar{r} = \bar{r}_i - \bar{r}_j (i \neq j)$. As seen from chapter1 by the current algebra approach [c.f. chapter 2], the amplitudes for non-leptonic decays of baryons are essentially determined by the matrix elements[2,5]

$$\left\langle B_s \middle| H_w^{p.c} \middle| B_r \right\rangle \sim \bar{u} a_{rs} u \tag{3.25}$$

where u is Dirac spinor of B_r or B_s which denotes a baryon like Λ, Σ, Ξ, n or p. Also, in the SU(3) space a_{rs} can be parametrized in terms of two reduced matrix elements D and F[2,5]

$$a_{rs} = \sqrt{2}(2Fif_{6rs} + 2Dd_{6rs}) \tag{3.26}$$

Now let us calculate the matrix element of the operator shown in Eq(3.20) between spin-unitary -spin wave functions of the octet baryons[2,14]. We obtain[11]

$$a_{\Lambda n} = \frac{G_F}{\sqrt{2}} \sin\theta_c \cos\theta_c d'(-\sqrt{6}) \tag{3.27a}$$

$$a_{\Sigma^+ p} = \frac{G_F}{\sqrt{2}} \sin\theta_c \cos\theta_c (-6)$$

$$= \sqrt{2} a_{\Sigma^0 n} \tag{3.27b}$$

$$a_{\Xi^0 \Lambda^0} = \frac{G_F}{\sqrt{2}} \sin\theta_c \cos\theta_c d'(-2\sqrt{6}) \tag{3.27c}$$

$$a_{\Xi^- \Sigma^-} = 0 \tag{3.27d}$$

The relation $a_{\Sigma^+p} = +\sqrt{2}a_{\Sigma^0n}$ expressed in Eq.(3.27b) supports the $\Delta I=1/2$ or octet dominance and leads to $A(\Sigma_+^+)=0$ in the current algebra approach(see Eq.(2.38)). From the parametrized parity-conserving matrix element, Eq.(3.26), and from the relation found in Eq.(3.27d), the quark model predicts the D/F ratio, which is an essential parameter in any current algebra approach[5], the following value[11]:

$$\frac{D}{F} = -1 \tag{3.28}$$

Now, using the relations expressed in Eq.(2.38) and using Eq.(3.26), we obtain, in addition to the Lee-Sugawara relation, the following relations[11]

$$2A\left(\Xi^- \to \Lambda\pi^-\right) - A\left(\Lambda^0 \to p\pi^-\right) = -\sqrt{3}A\left(\Sigma^+ \to p\pi^0\right) \tag{3.29a}$$

$$A\left(\Lambda \to p\pi^-\right) = -\frac{1}{\sqrt{3}}A\left(\Sigma^+ \to p\pi^0\right) \tag{3.29b}$$

$$A\left(\Xi^- \to \Lambda\pi^-\right) = -\frac{2}{\sqrt{3}}A\left(\Sigma^+ \to p\pi^0\right) \tag{3.29c}$$

From the parametrized matrix element,Eq.(3.26), and the relation Eq(3.29), we have

$$|a_{\Lambda n}| = \frac{1}{\sqrt{3}}|3F + D| = \frac{2}{\sqrt{3}}|F| \tag{3.30}$$

and equating the above result with relation Eq(3.27a), one can easily get[11]

$$|F|=\frac{G_F}{\sqrt{2}}\sin\theta_c\cos\theta_c\left(\frac{3}{\sqrt{2}}|d'|\right) \quad (3.31)$$

The d' can be estimated from the strong-interaction $\Delta - N$mass spliting to be[14]

$$d'=\frac{3(\Delta-N)}{8\pi\alpha_s}m_u^2$$

and this estimate gives[4]

$$|F|=\frac{\sin\theta_c\cos\theta_c}{16\alpha_s\pi}(\Delta-N)G_F\left(9m_u^2\right) \quad (3.32)$$

where $\alpha_S \approx 0.5$, $m_u \approx 0.34$GeV, $9m_u^2 \approx m_N^2$, $G_F(9m_u^2)=10^{-5}$, $\sin\theta_c\cos\theta_c \approx$ 0.221and Δ-N$\approx$300MeV, this gives |F| to be about 3×10^{-5} MeV.

The qurk model prediction of D/F= -1, d/f =3/2 and | F | $\approx 3\times10^{-5}$MeV do not provide a good fit of s- and p- wave amplitudes. However, a ratio of D/F=0.85, which is 15% less than the predicted value, together with d/f=1.8 would be in qualitative agreement with experiment fo p-wave amplitudes. On the other hand, fitting the p-wave amplitudes results in s-wave amplitudes large by a factor of ~ 2 as can be seen from[13] table 3.1. A sensible fit to both s-and p-waves was made by Gronau[15]. He pointed out that the problem of the D/F ratio for s-waves and the relative magnitude between s-and p-wave amplitudes can be cured by invoking a large correction to the soft pion limit for s-waves by including the so - called K*-pole term[13,15] (the corresponding Feynman diagram is shown in Fig.3.2. The K* contribution vanishes in the SU(3) limit that is the K* contribution to s-waves survive only because of SU(3) breaking in baryon masses.

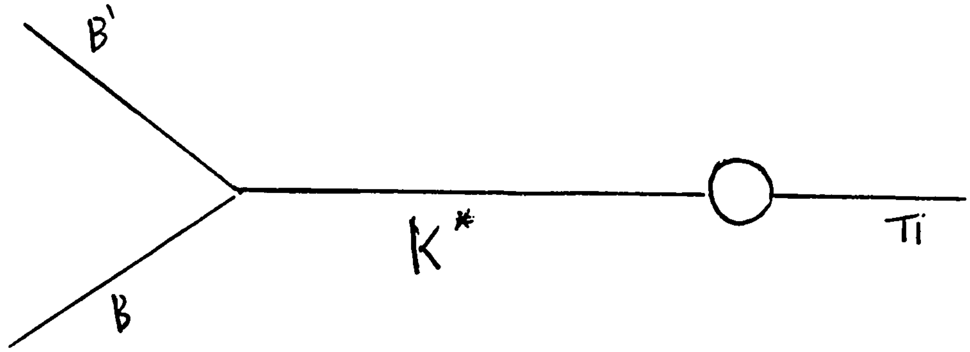


Fig. 3.2 : **Feynman diagram for the K^* contribution to hadronic decays of hyperons**

To see how the new contribution to s-wave amplitudes reduces the discrepancy between experimental results and the quark model prediction, let us start reviewing the general formula for the amplitude of the non-leptonic hyperon decays in the soft - pion limit.

$$B_j(p) \rightarrow B_k(p') + \pi_i(q)$$

The amplitude for hyperon decay of the type can be written by using the soft pion approximation in the following form[5]

$$T^{ijk}(q=0) = -\frac{\sqrt{2}}{F_\pi}\left\langle B_k \left\| \left[F_i^5(0), H(0) \right] \right\| B_j \right\rangle + \frac{\sqrt{2}}{F_\pi} \lim_{q\to 0} q^\mu M_\mu^{ijk} \tag{3.33}$$

where M_μ^{ijk} is called the surface term and is given by

$$M_\mu^{ijk} = \int d^4x e^{-iq.x} \left\langle B_k \left\| \left[A_{\mu i}(x), H(0) \right] \right\| B_j \right\rangle \theta(x_0) \tag{3.34}$$

The usual way of writting the amplitude T^{ijk} is by including the baryon pole term[5]

$$T^{ijk}(0) = B^{ijk}(q) + R^{ijk}(q) \tag{3.35}$$

where $B^{ijk}(q)$ is the baryon pole term. Now, from Eq.(3.33) we find[5]

$$R^{ijk}(q=0) = -\frac{\sqrt{2}}{F_\pi}\left\langle B_k \left\| \left[F_i^5(0), H(0)\right] \right\| B_j \right\rangle + \lim_{q\to 0}\left[\frac{\sqrt{2}}{F_\pi} q^\mu M_\mu^{ijk} - B^{ijk}(q)\right] \tag{3.36}$$

If one denotes the K*-pole contribution by K^{*ijk} one finds[15]

$$\begin{aligned} T^{ijk}\left(q^2 = m_\pi^2\right) = & \\ & \approx -\frac{\sqrt{2}}{F_\pi}\left\langle B_k \left\| \left[F_i^5(0), H(0)\right] \right\| B_j \right\rangle + K^{*ijk}\left(q^2 = m_\pi^2\right) \\ & + B^{ijk}\left(q^2 = m_\pi^2\right) + \lim_{q\to 0}\left[\frac{\sqrt{2}}{F_\pi} q^\mu M_\mu^{ijk} - B^{ijk}(q)\right] \end{aligned} \tag{3.37}$$

while[5]

$$K^{*ijk} = c d_{i6l}\left(i f_{lkj} + \frac{\delta}{\phi} d_{lkj}\right)\left(m_j - m_k\right)\bar{u}(p')u(p) \tag{3.38}$$

where δ/ϕ is the D/F ratio for the γ_μ coupling at the strong VBB vertex and it has been assumed that K^* contribution satisfies $\Delta I = 1/2$ or octet rule. The constant $c = 3.2 \times 10^{-9}$ MeV^{-1} which has been obtained by fitting the $K \to 2\pi$ decay amplitude from K^*-pole only (a questionable assumption). The s-wave amplitudes after this contribution is added, would be[15]

$$
\begin{aligned}
&A\left(\Sigma_{-}^{-}\right)=\left(\frac{\sqrt{2}}{F_{\lambda}}\right)(-D+F)-\frac{1}{4}c\sqrt{2}(\Sigma-N)\left(1-\frac{\delta}{\phi}\right)\\
&A\left(\Sigma_{+}^{+}\right)=0\\
&A\left(\Sigma_{0}^{+}\right)=F_{\pi}^{-1}(D-F)+\frac{1}{4}c(\Sigma-N)\left(1-\frac{\delta}{\phi}\right)\\
&A\left(\Lambda_{-}^{0}\right)=\left(\sqrt{3}F_{\pi}\right)^{-1}(D+F)-\frac{1}{4}c\sqrt{3}(\Lambda-N)\left[1+\frac{1}{2}\left(\frac{\delta}{\phi}\right)\right]\\
&A\left(\Xi_{-}^{-}\right)=\left(\sqrt{3}F_{\pi}\right)^{-1}(D-3F)+\frac{1}{4}c\sqrt{3}(\Xi-\Lambda)\left[1-\frac{1}{3}\left(\frac{\delta}{\phi}\right)\right]
\end{aligned}
\tag{3.39}
$$

The best fit to the experimental amplitides is obtained with the following values[15]

$|F| = 4.7 \times 10^{-5} MeV^2$

$D/F=-0.85$

$d/f=1.8$

$\delta/\phi=-0.5$

Table 3.1 sumarizes the results of the best fit and are compared with the corresponding experimental values[7].

The problem of the s-wave amplitudes was also treated by considering an other correction which comes from the first negative parity level of the baryon spectrum ($\underline{70}$, 1^-) and it simulates the K* type contributon.The contribution satisfies the $\Delta I=1/2$ rule and vanishes in the soft pion limit[13]. By this correction, we get a significant improvement of the relative magnitude of s-and p- waves . Denoting the new contribution by A_i^{70}, we have the following expression for the s-waves[13]

$$A_i = -\frac{\sqrt{2}}{F_\pi}(2\pi)^3\sqrt{\frac{p_B^0 p_{B'}^0}{m_B m_{B'}}}\left\langle B'\left|\left[F_i, H_w^{pc}(0)\right]\right|B\right\rangle + A_i^{(70)} \tag{3.40}$$

$$A_i^{(70)} = -\frac{\sqrt{2}}{F_\pi}(2\pi)^3\sqrt{\frac{p_B^0 p_{B'}^0}{m_B m_{B'}}}\,q_\mu \sum_{(\underline{70},1^-)}\left\{\frac{\langle B'|A_\mu^i(0)|n\rangle\langle n|H_w^{pv}(0)|B\rangle}{p_{B'}^0 - p_n^0 + q^0} + \frac{\langle B'|H_w^{pv}(0)|n\rangle\langle n|A_\mu^i(0)|B\rangle}{p_B^0 - p_n^0 - q^0}\right\} \tag{3.41}$$

All the matrix elements of H_W and of the axial current are computed in the exact SU(3) limit and in quark model . Also, the harmonic potential is used in the computation of the ($\underline{70}$.1^-) contribution. Table 3.1 summarizes all the results obtained by invoking corrections to s-wave amplitudes[13].

TABLE 3. 1 **s-and p- waves in the commutator, $\frac{1}{2}^+$ baryon pole approximation and taking into account the correction to the soft pion limit given by the (70,1^-) intermediate states and the correction given by the K^* - pole contribution.**

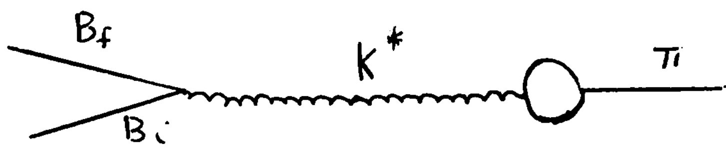
	s-wave 10^6 A					p-wave 10^6 B		
	D/F		K^*pole	$(\underline{70},1^-)$	Expt.	D/F = −1	D /F = −0.85	Expt.
Transition	−1	−0.85	correction	correction		d /f = 1.5	d /f = 1.8	
Σ^-_-	0.998	0.911	0.456	0.434	0.427 ± 0.002	0.868	−0.087	−0.14 ± 0.02
Σ^+_0	−0.716	−0651	− 0.326	−0.326	−0.326 ± 0.011	2.80	3.168	2.67 ± 0.15
Σ^+_+	0	0	0	0	0.014 ± 0.003	4.817	4.405	4.22 ± 0.01
Λ^0_-	0.412	0.434	0.260	0.303	0.323 ± 0.002	1.953	2.517	2.20 ± 0.05
Ξ^-_-	− 0.825	−0.781	− 0.434	− 0.477	− 0450 ± 0.002	1.758	1.671	1.75 ± 0.06

CHAPTER 4

Calculation of the K^*-Pole Weak Transition in Non-Relativistic Quark Model

We have seen in the previous chapter taht a good fit to both s- and p-wave amplitudes can be obtained if two possible corrections, namely K^*-pole or $(70,1^-)$ baryon pole contributions are added to the s- wave amplitudes obtained from current algebra together with the quark model used to calculate $\langle B_s | H_W^{p.c} | B_r \rangle$ which also appear in p-wave amplitudes. While $(70,1^-)$ contribution has been calculated on the quark model[13] that of the K^* has not been calculated in this model. To be consistent with the approach being followed, one should also calculate K^* contribution in the quark model. This would also shed some light, which of the two possible contributions mentained above would dominate.

The K^* pole contribution to the s-wave non-leptonic decays of hyperons is depicted in the diagram of Fig. 4.1



$$: \frac{g_{B_iB_fK^*}\, b_{K^*\pi}}{m^2_{K^*}}$$

Fig.4.1 : Feynman diagram for K^* contribution to hadronic decays of hyperons

This contribution involves the strong coupling constant $g_{B_iB_fK^*}$ and the weak transition amplitude $b_{K^*\pi}$ defined by

$$\sqrt{2E_{K^*}2E_{\pi^0}}\left\langle\pi\left|H_W^{p.v}\right|K^*\right\rangle = P_\pi.\varepsilon_\mu b_{K^*\pi} \tag{4.1}$$

where ε_μ is the polarization vector of K^* - meson. The strong coupling constant g can be estimated through the vector meson dominance of the matrix element

$\left\langle B_f\left|V_{\ell\mu}\right|B_i\right\rangle$ as shown in Fig.4.2, where $V_{\ell\mu}\,(\ell = 1...8)$ is octet of vector current:

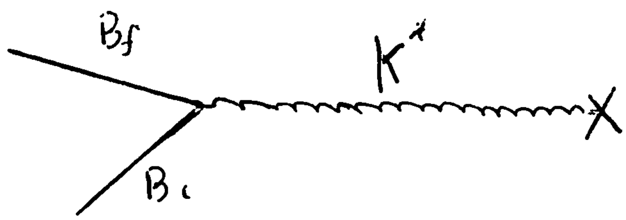


Fig.4.2 : **The vector meson dominance as can be seen from the above figure**

$$\left\langle 0\left|V_{\ell\mu}\right|K_k^*\right\rangle = \frac{f_{K^*}}{\sqrt{2}}\varepsilon_\mu\delta_{\ell\mu} \tag{4.2}$$

This diagram gives

$$\frac{g_{B_iB_fK^*}}{\sqrt{2}m_{K^*}^2}f_{K^*} = if_{kif} \tag{4.3}$$

Thus for example for ΛpK^{*-}, we have

$$\frac{g_{\Lambda pK^{*-}} f_{K^*}}{\sqrt{2}m_{K^*}^2} = if_{\frac{4-i5}{\sqrt{2}}\frac{4+i5}{\sqrt{2}}8} = -\sqrt{\frac{3}{2}} \tag{4.4}$$

Thus the K^* contribution to $\Lambda \to p\pi^-$ is

$$-\sqrt{\frac{3}{2}}\frac{b_{K^{*-}\pi^-}}{f_{K^*}} \tag{4.5}$$

Including the K^* contribution, we thus obtain

$$A\left(\Lambda_-^0\right) = \frac{1}{\sqrt{3}f_\pi}(D+3F) - \sqrt{\frac{3}{2}}\frac{b_{K^{*-}\pi^-}}{f_{K^*}}(\Lambda - N)$$

$$A\left(\Lambda_0^0\right) = -\frac{1}{\sqrt{6}f_\pi}(D+3F) - \sqrt{\frac{3}{2}}\frac{b_{\overline{K}^{*0}\pi^0}}{f_{K^*}}(\Lambda - N)$$

$$A\left(\Sigma_0^+\right) = \frac{1}{f_\pi}(D-F) - \frac{b_{\overline{K}^{*0}\pi^0}}{f_{K^*}}(\Sigma - N)$$

$$A\left(\Sigma_+^+\right) = 0$$

$$A\left(\Sigma_-^-\right) = -\frac{\sqrt{2}}{f_\pi}(D-F) - \frac{b_{K^{*-}\pi^-}}{f_{K^*}}(\Sigma - N)$$

$$A\left(\Xi_-^-\right) = \frac{1}{\sqrt{3}f_\pi}(D-3F) + \sqrt{\frac{3}{2}}\frac{b_{K^{*-}\pi^-}}{f_{K^*}}(\Xi - \Lambda)$$

$$A\left(\Xi_0^0\right) = -\frac{1}{\sqrt{6}f_\pi}(D-3F) + \sqrt{\frac{3}{2}}\frac{b_{\overline{K}^{*0}\pi^0}}{f_{K^*}}(\Xi - \Lambda)$$

(4.6)

where $\frac{b_{K^*\pi}}{f_{K^*}}$ was treated as a free parameter by Gronau in his fit for hyperon decays.

Now we are going to calculate the $\bar{K}^{*0}$-π^0 transition in the non-relativistic quark model in similar way described in the last chapter. The spin and flavor wave functions for $\bar{K}^{*0}$ and π^0 are given below

$$\left|\bar{K}^{*0}\right\rangle = \frac{1}{\sqrt{2}}\left(\bar{d}^{\uparrow}s^{\downarrow} + \bar{d}^{\downarrow}s^{\downarrow}\right)$$

$$\left|\pi^0\right\rangle = \frac{1}{2}\left(\bar{u}^{\uparrow}u^{\downarrow} - \bar{u}^{\downarrow}u^{\uparrow} - \bar{d}^{\uparrow}d^{\downarrow} + \bar{d}^{\downarrow}d^{\uparrow}\right) \tag{4.7}$$

The K^*-π transition is described by the quark-antiquark scattering through the exchange of W^+ boson as shown in Figure 4.3.

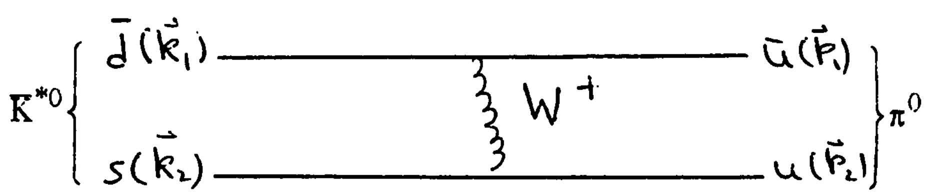


Figure 4.3 : **The quark-antiquark scattering diagram for $\bar{K}^{*0}$ - π^0 transition.**

The Hamiltonian which describes the above process has the following form

$$\frac{g_w^2}{m_w^2}\sin\theta_c\cos\theta_c\left[\bar{u}(\bar{p}_2)\gamma_\mu(1+\gamma_5)\alpha_1^- u(\bar{k}_2).\beta_2^+\bar{\upsilon}(\bar{k}_1)\gamma_\mu(1+\gamma_5)\upsilon(\bar{p}_1)\right] \tag{4.8}$$

We are interested in the parity violating part of the above matrix element which is given by the following expression

$$\bar{u}(\bar{p}_2)\gamma\gamma_5 u(\bar{k}_2).\bar{\upsilon}(\bar{k}_1)\gamma\upsilon(\bar{p}_1)+\bar{u}(\bar{p}_2)\gamma_4\gamma_5 u(\bar{k}_2).\bar{\upsilon}(\bar{k}_1)\gamma_4\upsilon(\bar{p}_1)$$
$$+\bar{u}(\bar{p}_2)\gamma u(\bar{k}_2).\bar{\upsilon}(\bar{k}_1)\gamma\gamma_5\upsilon(\bar{p}_1)+\bar{u}(\bar{p}_2)\gamma_4 u(\bar{k}_2).\bar{\upsilon}(\bar{k}_1)\gamma_4\gamma_5\upsilon(\bar{p}_1) \quad (4.9)$$

where u's and υ's are given by

$$u^r(\bar{p})=\frac{1}{\sqrt{2m(E+m)}}\begin{pmatrix}E+m\\ \bar{\sigma}.\bar{p}\end{pmatrix}\chi^r \quad (4.10a)$$

$$\upsilon^r(\bar{p})=\frac{1}{\sqrt{2m(E+m)}}\begin{pmatrix}\bar{\sigma}.\bar{p}\\ E+m\end{pmatrix}\chi^r \quad (4.10b)$$

Using the non - relativistic expansion in Eq.(4.2) i.e. by keeping only the leading order terms one gets

$$\bar{u}(\bar{p}_2)\gamma u(\bar{k}_2)\approx\frac{-i}{2m}\left[(\bar{p}_2+\bar{k}_2)+i\bar{\sigma}_2\times(\bar{p}_2-\bar{k}_2)\right]$$
$$\bar{u}(\bar{p}_2)\gamma_4 u(\bar{k}_2)$$
$$\bar{u}(\bar{p}_2)\gamma\gamma_5 u(\bar{k}_2)\approx i\bar{\sigma}_2$$
$$\bar{u}(\bar{p}_2)\gamma_4\gamma_5 u(\bar{k}_2)\approx-\frac{\bar{\sigma}_2}{2m}(\bar{p}_2+\bar{k}_2) \quad (4.11)$$
$$\bar{\upsilon}(\bar{k}_1)\gamma\upsilon(\bar{p}_1)\approx-\frac{i}{2m}\left[(\bar{k}_1+\bar{p}_1)+i\bar{\sigma}\times(\bar{k}_1-\bar{p}_1)\right]$$
$$\bar{\upsilon}(\bar{k}_1)\gamma_4\upsilon(\bar{p}_1)\approx 1$$
$$\bar{\upsilon}(\bar{k}_1)\gamma\gamma_5\upsilon(\bar{p}_1)\approx i\bar{\sigma}_1$$
$$\bar{\upsilon}(\bar{k}_1)\gamma_4\gamma_5\upsilon(\bar{p}_1)\approx-\frac{\bar{\sigma}_1}{2m}.(\bar{k}_1+\bar{p}_1) \quad (4.12)$$

Then in the non-relativistic limit $H_w^{p.v}$ takes the form

$$H_w^{p.v} \approx -\frac{G_F}{\sqrt{2}} \sin\theta_c \cos\theta_c \frac{1}{2m}\Big[(\vec{\sigma}_2 - \vec{\sigma}_1).\{(\vec{k}_1 + \vec{p}_1) - (\vec{p}_2 + \vec{k}_2)\} + i\{(\vec{k}_1 - \vec{p}_1) - (\vec{p}_2 - \vec{k}_2)\}.(\vec{\sigma}_2 \times \vec{\sigma}_1)\Big]\alpha_1^- \beta_2^+ \quad (4.13)$$

where m is the averge constituent quark mass $m = \frac{m_u + m_s}{2}$. In order to evaluate $\langle \pi^0 | H_w^{p.v} | K^{*0} \rangle$, we need to consider only the z-components of the spin operators

$$(\vec{\sigma}_2 \times \vec{\sigma}_1)_{\hat{z}} \alpha_1^- \beta_2^+ |K^{*0}\rangle_{\text{flavor,spin}} = \frac{2i}{\sqrt{2}}\left\{\bar{u}^{\uparrow} u^{\downarrow} - \bar{u}^{\downarrow} u^{\uparrow}\right\} \quad (4.14a)$$

$$(\vec{\sigma}_2 - \vec{\sigma}_1)_{\hat{z}} \alpha_1^- \beta_2^+ |K^{*0}\rangle_{\text{flavor,spin}} = -\frac{2}{\sqrt{2}}\left\{\bar{u}^{\uparrow} u^{\downarrow} - \bar{u}^{\downarrow} u^{\uparrow}\right\} \quad (4.14b)$$

To proceed further we introduce relative and center of mass momenta:

$$\vec{k}_i = \vec{k}_1 - \vec{k}_2 \quad , \quad \vec{K}_{K^{*0}} = \vec{k}_1 + \vec{k}_2$$
$$\vec{p}_i = \vec{p}_1 - \vec{p}_2 \quad , \quad \vec{P}_{\pi^0} = \vec{p}_1 + \vec{p}_2$$

Now, applying the parity violating Hamiltonian given in Eq.(4.13) and by using the results obtained in Eq.(4.14a) and Eq.(4.14b), one gets

$$H_w^{p.v}|K^{*0}\rangle = \frac{G_F}{\sqrt{2}} \frac{\sin\theta_c \cos\theta_c}{2m} \int \phi_{K^{*0}}(\vec{k}_i)\Big[-\frac{2}{\sqrt{2}}\{(\vec{k}i - \vec{p}_i)\} - \frac{2i}{\sqrt{2}}\{(\vec{K}_{K^{*0}} - \vec{P}_{\pi^0})\}\left\{\bar{u}^{\uparrow} u^{\downarrow} - \bar{u}^{\downarrow} u^{\uparrow}\right\}\Big] d\vec{k}_1 d\vec{k}_2 \delta(\vec{k}_1 + \vec{k}_2 - \vec{K}_{K^{*0}}) \quad (4.15)$$

Then we obtain (noting that in the non-relativistic limit $E_{K^*} \approx m_{K^*}, E_\pi \approx m_\pi$)

$$\sqrt{4E_{K^*}E_\pi}\left\langle \pi^0 \left| H_w^{p.v} \right| K^{*0} \right\rangle$$
$$= \frac{G_F}{\sqrt{2}} \sin\theta_c \cos\theta_c \sqrt{2m_{K^*} 2m_\pi} \frac{2}{\sqrt{2}(2m)} \bar{\varepsilon}. \int \phi^*_{\pi^0}(\bar{p}_i)\phi_{K^{*0}}(\bar{k}_i) \times$$
$$\left\{ (\bar{k}_i - \bar{p}_i) + (\bar{K}_{K^{*0}} - \bar{P}_{\pi^0}) \right\} d\bar{p}_i d\bar{k}_i \tag{4.16}$$

By taking the Fourier transform in terms of the spatial wave functions in configuration space, the right hand side of Eq(4.16) becomes

$$= -\frac{G_F}{\sqrt{2}} \sin\theta_c \cos\theta_c \sqrt{2m_{K^*} 2m_{\pi^0}} \frac{2}{\sqrt{2}(2m)} i\bar{\varepsilon}$$
$$\times (2\pi)^{-6} \int \phi^*_{\pi^0}(\bar{r}')\phi_{K^{*0}}(\bar{r}) \left\{ (\bar{k}_i - \bar{p}_i) + (\bar{K}_{K^{*0}} - \bar{P}_{\pi^0}) \right\} e^{-ip.r} e^{i\bar{k}_i.\bar{r}} d\bar{r} d\bar{r}' d\bar{p}_i d\bar{k}_i \tag{4.17}$$

If we let

$$\bar{k}_i \to -\bar{k}_i \qquad \bar{r} \to -\bar{r}$$
$$\bar{p}_i \to -\bar{p}_i \qquad \bar{r}' \to -\bar{r}'$$

then $\phi^*_{\pi^0}(\bar{r}_i')\phi_{K^{*0}}(\bar{r}_i)$being spherically symmetric s-wave functions are unchanged while the first term in Eq.(4.17) changes sign. This implies that the first term of Eq.(4.17) is just zero. So Eq.(4.17) becomes

$$\sqrt{2K_{\bar{K}^{*0}}2P_{\pi^0}}\left\langle \pi^0 \middle| H_w^{p.v} \middle| \bar{K}^{*0} \right\rangle = -\frac{G_F}{\sqrt{2}}\sin\theta_c\cos\theta_c\sqrt{2m_{\bar{K}^{*0}}2m_{\pi^0}}\frac{2}{\sqrt{2}(2m)}$$
$$\times\bar{\varepsilon}.\left(K_{\bar{K}^{*0}}-P_{\pi^0}\right)\phi^*_{\pi^0}(0)\phi_{\bar{K}^{*0}}(0) \qquad (4.18)$$

Hence finally, comparing with Eq.(4.1) we obtain

$$b_{\bar{K}^{*0}\pi^0} = \frac{G_F}{\sqrt{2}}\sin\theta_c\cos\theta_c\sqrt{2m_{\bar{K}^{*0}}2m_{\pi^0}}\frac{2}{\sqrt{2}(2m)}\phi^*_{\pi^0}(0)\phi_{\bar{K}^{*0}}(0) \qquad (4.19)$$

The $K^{*-}-\pi^-$ transition is given by the annhilation diagram

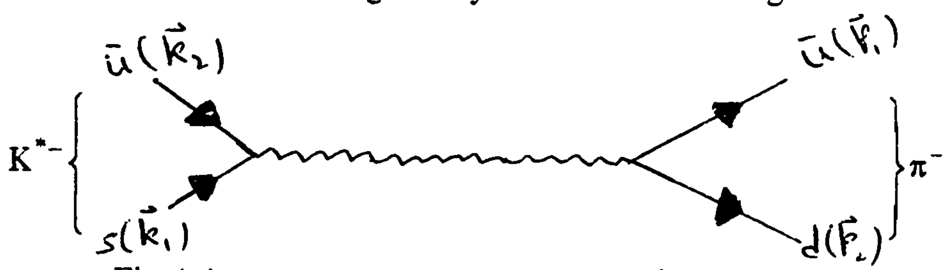


Fig.4.4 : **The annhilation diagram for K^*-π transition.**

This diagram gives the matrix element

$$3\frac{g_w^2}{m_w^2}\sin\theta_c\cos\theta_c\alpha_1^-\beta_2^+\left[\bar{u}(\bar{p}_2)\gamma_\mu(1+\gamma_5)\upsilon(\bar{p}_1).\bar{\upsilon}(\bar{k}_1)\gamma_\mu(1+\gamma_5)u(\bar{k}_2)\right] \quad (4.20)$$

where 3 is the color factor. Now if we use the Fierz rearrangment theorem[5], this reduces to

$$3\frac{g_w^2}{m_w^2}\sin\theta_c\cos\theta_c\alpha_1^-\beta_2^+\left[\bar{u}(\bar{p}_2)\gamma_\mu(1+\gamma_5)u(\bar{k}_2).\bar{\upsilon}(\bar{k}_1)\gamma_\mu(1+\gamma_5)\upsilon(\bar{p}_1)\right] \quad (4.21)$$

i.e. it reduces to Eq.(4.1) except for the color factor 3. Thus

$$b_{K^{*-}\pi^-} = 3\sqrt{2}b_{\bar{K}^{*0}\pi^0} \tag{4.22}$$

the factor $\sqrt{2}$ comes from the fact that $|\pi^-\rangle = |\bar{u}d\rangle$ while $|\pi^0\rangle = [\bar{u}u - \bar{d}d]$ as far as the flavor is concerned. To find a numerical estimate of $\frac{b_{\bar{K}^{*0}\pi^0}}{f_{K^*}}$ which appear in Eq.(3.6) for $A(\Lambda^0_-)$ etc, we note that in the SU(3) limit[16] [$f_\pi \approx 0.14$GeV]

$$f_{K^*} \approx f_\rho \approx \sqrt{2}m_\rho f_\pi \approx 0.14\text{GeV}^2 \tag{4.23}$$

and that $\phi^*_{\pi^0}(0)$and $\phi_{\bar{K}^{*0}}(0)$ as determined from the leptonic decay widths of ρ and K^* are given by[17]

$$|\phi_{\pi^0}(0)|^2 \approx |\phi_\rho(0)|^2 \approx 0.29\times 10^{-2}\,\text{GeV}^3$$

$$|\phi_{K^*}(0)|^2 \approx 0.385\times 10^{-2}\,\text{GeV}^3 \tag{3.24}$$

Thus from Eq.(3.19)

$\left[\text{with } G_F \approx 10^{-5}\text{GeV}^{-2},\ \sin\theta_c\cos\theta_c \approx 1/4,\ 2m \approx m_{K^*} \approx 890\text{MeV}\right]$, we obtain

$$\frac{b_{\bar{K}^{*0}\pi^0}}{f_{K^*}} \approx 0.47\times 10^{-10}\text{MeV}^{-1} \tag{4.25}$$

This estimate is smaller by an order of magnitude than what Gronu needed for his fit. This in fact is gratifing since the relation (4.22) obtained in quark model badly violates the $\Delta I=1/2$ rule, which requires

$$b_{K^{*-}\pi^-} = -\sqrt{2}\, b_{\overline{K}^{*0}\pi^0} \tag{4.26}$$

The $\Delta I=1/2$ rule is satisfied by non-leptonic hyperon decays to a very good approximation. If we define

$$\Delta(\Lambda) = \frac{A\left(\Lambda^0_-\right) + \sqrt{2}A\left(\Lambda^0_0\right)}{A\left(\Lambda^0_-\right)} \tag{4.27}$$

etc. then the $\Delta I=1/2$ rule requires that $\Delta(\Lambda)=0$. The experimental situation is summarized[7] in Table 4.1.

TABLE 4.1: **comparison of the observed deviation from $\Delta I=1/2$ rule for s-wave hyperon decay amplitude and the estimated for K^*-π transition**

	Observed	Estimated
$\Delta(\Lambda)$	-(3.7±1.8)%	-7.7%
$\Delta(\Xi)$	-(5.73±1.65)%	-5.2%
$\Delta(\Sigma)$	(11.2±3.5)%	6.9%

As seen above, K^*-π transition does violate $\Delta I = 1/2$ rule. Before we compare this contribution to $\Delta(\Lambda)$ etc. with its experimental value, we wish to make two remarks: the QCD renormalization[18] of weak interactions necessates that $b_{\bar{K}^{*0}\pi^0}$ contribution in Eq.(4.19) should be multiplied by a factor (C_2+3C_1) so that

$$b_{\bar{K}^{*0}\pi^0} = (C_2 + 3C_1)\frac{G_F}{\sqrt{2}}\sin\theta_c\cos\theta_c\sqrt{2m_{\bar{K}^{*0}}\,2m_{\pi^0}}$$

$$\times\frac{2}{\sqrt{2}(2m)}\phi^*_{\pi^0}(0)\phi_{\bar{K}^{*0}}(0) \qquad (4.28)$$

and that for $b_{K^{*-}\pi^-}$ is

$$b_{K^{*-}\pi^-} = (3C_2 + C_1)\sqrt{2}\frac{G_F}{\sqrt{2}}\sin\theta_c\cos\theta_c\sqrt{2m_{K^{*-}}\,2m_{\pi^-}}$$

$$\times\frac{2}{\sqrt{2}(2m)}\phi^*_{\pi^0}(0)\phi_{\bar{K}^{*0}}(0) \qquad (4.29)$$

In the absence of QCD radiative corrections, $C_2 = 1$ while $C_1 = 0$ while in the presence of these corrections[18]

$$C_1 = -0.587 \quad , \quad C_2 = 1.319 \qquad (4.30)$$

so that

$$(C_2+3C_1) \approx -0.44 \quad , \quad (3C_2+C_1) \approx 3.37 \qquad (4.31)$$

Thus using Eq.(4.26) and Eq.(4.31)

$$\frac{b_{K^{*-}\pi^-} + \sqrt{2}b_{\bar{K}^{*0}\pi^0}}{f_{K^*}} \approx 4.144(0.47) \times 10^{-10}\,\mathrm{MeV}^{-1}$$
$$\approx 1.95 \times 10^{-10}\,\mathrm{MeV}^{-1} \tag{4.32}$$

The second remark concerns that K^* pole is highly off mass shell since $K^2_{K^*} = P^2_\pi \approx m^2_\pi$. To take care of the off mass shell behaviour of K^*, we use the recipe to multiply the contribution in Eq.(4.28) and Eq.(4.29) by the factor[19]

$$\frac{m^2_{K^{*\prime}} - m^2_{K^*}}{m^2_{K^{*\prime}} - q^2} \tag{4.33}$$

where $K^{*\prime}$ is the radial excitation of K^* and has the mass $m_{K^{*\prime}} \approx 1.4\mathrm{GeV}$. This factor reduces to 1 when K^* is on mass shell i.e. $q^2 = m^2_{K^*}$. We observe that the recipe of Eq.(4.33) satisfies both the QCD counting rules and the asymptotic $q^2 \to \infty$ behavior of K^*-pole dominated dispersion relation in q^2. For $q^2 = m^2_\pi$, this gives the reduction factor 0.6 so that the estimate (4.32) reduces to

$$\frac{b_{K^{*-}\pi^-} + \sqrt{2}b_{\bar{K}^{*0}\pi^0}}{f_{K^*}} \approx 1.17 \times 10^{-10}\,\mathrm{MeV}^{-1} \tag{4.34}$$

Finally if we make use of Eq.(4.33) and Eq.(4.34), we can calculate $\Delta(\Lambda)$ etc. . These are summarized in Table 4.1.

We see that the K^{*}-π contribution gives the right order of magnituide for the experimental deviations from the $\Delta I=1/2$ rule.

We also conclude that K^{*}-π contribution to non-leptonic decays which satisfies the $\Delta I=1/2$ to a very good approximation, is negligible and can not be responsible for the fit obtained by Gronu. This would indicate that one should look for other contributions, like those of $(70,1^{-})$ baryon poles to obtain a fit to the non - leptonic decay of hyperons, discussed in the previous chapter.

CHAPTER 5

Conclusions

The non-leptonic decays of hyperons as presented in Eq(2.15) involve combinations of s-and p-waves which are represented by the constant amplitudes A and B respectively, where A stand for the parity violating part (s-wave) and B stand for the parity conserving part (p-wave). This is a fact which has been drawn from the conservation of angular momentum. The weak interactions of the hadronic decays are governed by a general rule observed experimentally, the $\Delta I=1/2$ rule. However, the current-current form of weak interactions contain both $\Delta I=1/2$ and $\Delta I=3/2$ where the $\Delta I=1/2$ is enhanced and $\Delta I=3/2$ is suppressed, this is known in the language of SU(3) representation as the octet dominance.

The current -current picture of the weak Hamiltonian when applied to the non-leptonic decays involves complications due to the strong interactions when the matrix element of the hamiltonian is taken between relevant hadronic states. But these complictions have been fairly solved by that of current algebra and partial conservation of axial vector approach. By this approach, one can show that within SU(3) limit that there is no parity violating transition betwen two baryons, is

$\langle B_s(p')|H_W^{p.v}|B_r(p)\rangle \approx 0$. In the current algebra approach the s-andp-wave amplitudes are determined by the weak matrix element

$$\langle B_s(p')|H_W^{p.c}|B_r(p)\rangle \tag{5.1}$$

where B_S and B_r are members of octet baryon. The above matrix elements can be parametrized in terms of the reduced matrix element F and D

$$a_{rs} = \sqrt{2}(2Fif_{6rs} + 2Dd_{6rs}) \tag{5.2}$$

where the s-and p-wave amplitudes have been expressed in terms of them as shown in Eq (2.39) and Eq(2.40). The quark model predicts the D/F=-1, and $|F| \approx 3\times10^{-5}$MeV. On the other hand, the experimental results of the s-and p-wave amplitudes can not be simultaneously fitted with the quark model prediction as shown in table 3.1. However, a ratio of D/F=0.85, which is 15% less than the predicted one, would fix the p-wave amplitudes but would increase the discrepancy in the s-wave amplitudes by a factor of ~2 as can be seen from table 3.1.

The problem of s-wave amplitudes has been treated by two methods:
1) the K^*-pole contribution considered by Gronau[15].
2) the contribution from the first negative parity level of the baryon spectrum (70, 1^-) made by A.Le Yaouanc et al.[13].

The Gronu fit involved a parameter c [c.f.Eq.(3.37)] which was treated as free parameter and was fixed without any theory. We have calculated the c parameter by using quark model as can be seen from Eq.(4.13) and it turns out that it is small and also it dose not satisfy the $\Delta I=1/2$ rule and can not be responsible for the fit obtained by Gronu. But it is interesting to note that this contribution as calculated in chapter 3 on quark model gives the right order of magnitude of observed deviation from the $\Delta I=1/2$ rule. On the contrary, the second approach which considered the correction from the first negative parity level of the baryon spectrum $(70, 1^-)$ dose satisfy the $\Delta I=1/2$ rule. This correction vanishes in the soft pion limit and it adds a signifcant improvement to the fit of s- and p- wave amplitudes to the non-leptonic decays of hyperons (see table 3.1).

Appendix A
Matrix Element

In the construction of the hamiltonian Eq(4.6), we have omitted some details of the calculation. Here we give some details of the expansion and approximation that are required in the construction of the matrix element. Eq(4.3)and Eq(4.4) have originally the following form just before the non -relativistic expansion was made:

$$
\begin{aligned}
&\bar{u}(\bar{p}_2)\gamma u(\bar{k}_2)=\frac{-i}{2m}\left[A\bar{\sigma}_2(\bar{\sigma}_2.\bar{k}_2)+A^{-1}(\bar{\sigma}_2.\bar{p}_2)\bar{\sigma}_2\right]\\
&\bar{u}(\bar{p}_2)\gamma_4 u(\bar{k}_2)=\frac{1}{2m}\left[B+B^{-1}(\bar{\sigma}_2.\bar{p}_2)(\bar{\sigma}_2.\bar{k}_2)\right]\\
&\bar{u}(\bar{p}_2)\gamma\gamma_5 u(\bar{k}_2)=\frac{i}{2m}\left[B\bar{\sigma}_2+B^{-1}(\bar{\sigma}_2.\bar{p}_2)\bar{\sigma}_2(\bar{\sigma}_2.\bar{k}_2)\right]\\
&\bar{u}(\bar{p}_2)\gamma_4\gamma_5 u(\bar{k}_2)=\frac{-1}{2m}\left[A^{-1}(\bar{\sigma}_2.\bar{p}_2)+A(\bar{\sigma}_2.\bar{k}_2)\right]
\end{aligned}
\tag{A.1}
$$

$$
\begin{aligned}
&\bar{\upsilon}(\bar{k}_1)\gamma\upsilon(\bar{p}_1)=\left[A(\bar{\sigma}_1.\bar{k}_1)\bar{\sigma}_1+A^{-1}\bar{\sigma}_1(\bar{\sigma}_1.\bar{p}_1)\right]\\
&\bar{\upsilon}(\bar{k}_1)\gamma_4\upsilon(\bar{p}_1)=\left[B+B^{-1}(\bar{\sigma}_1.\bar{k}_1)(\bar{\sigma}_1.\bar{p}_1)\right]\\
&\bar{\upsilon}(\bar{k}_1)\gamma\gamma_5\upsilon(\bar{p}_1)=\frac{i}{2m}\left[B\bar{\sigma}_1+B^{-1}(\bar{\sigma}_1.\bar{k}_1)\bar{\sigma}_1(\bar{\sigma}_1.\bar{p}_1)\right]\\
&\bar{\upsilon}(\bar{k}_1)\gamma_4\gamma_5\upsilon(\bar{p}_1)=\frac{-1}{2m}\left[A(\bar{\sigma}_1.\bar{k}_1)+A^{-1}(\bar{\sigma}_1.\bar{p}_1)\right]
\end{aligned}
\tag{A.2}
$$

where $A=\sqrt{\dfrac{E'+m}{E+m}}$, $B=\sqrt{(E'+m)(E+m)}$. Now, we are going to perform several approximations and expansions to the above terms

$$(E+m)^{1/2}=\left(2m+\frac{p^2}{2m^2}\right)\approx\sqrt{2m}\left(1+\frac{p^2}{8m^2}\right)$$

$$\frac{1}{E}\approx\frac{1}{m}\left(1-\frac{p^2}{2m^2}\right) \qquad ,\frac{1}{\sqrt{E}}\approx\frac{1}{\sqrt{2m}}\left(1-\frac{p^2}{4m^2}\right)$$

$$(E+m)^{-1/2}\approx\frac{1}{\sqrt{2m}}\left(1-\frac{p^2}{8m^2}\right)$$

$$A\approx 1+\frac{p'^2}{8m^2}-\frac{p^2}{8m^2} \qquad , \qquad A^{-1}\approx 1-\frac{p'^2}{8m^2}+\frac{p^2}{8m^2}$$

$$B\approx 2m\left(1+\frac{p^2}{8m^2}+\frac{p'^2}{8m^2}+---\right) \qquad B^{-1}\approx\frac{1}{2m}\left(1-\frac{p^2}{8m^2}-\frac{p'^2}{8m^2}----\right)$$

By keeping only linear terms in $\frac{|p|}{m}$ we get

$$A=A^{-1}\approx 1 \quad , \quad B\approx 2m, \ B^{-1}\approx 1/2m \tag{A.3}$$

Now , we use the following identities

$$\begin{aligned}
&\vec{\sigma}(\vec{\sigma}.\vec{p})=\vec{p}-i(\vec{\sigma}\times\vec{p})\\
&(\vec{\sigma}.\vec{p}')\vec{\sigma}=\vec{p}'+i(\vec{\sigma}\times\vec{p}')\\
&(\vec{\sigma}.\vec{p}')(\vec{\sigma}.\vec{p})=\vec{p}'.\vec{p}+i\vec{\sigma}.(\vec{p}'\times\vec{p})\\
&(\vec{\sigma}.\vec{p}')\vec{\sigma}(\vec{\sigma}.\vec{p})=(\vec{\sigma}.\vec{p})\vec{p}'-i(\vec{p}'\times\vec{p})-\vec{\sigma}(\vec{p}'.\vec{p})+(\vec{\sigma}.\vec{p}')\vec{p}
\end{aligned} \tag{A.4}$$

By performing such approximations and expansions and by considering only linear terms , one can construct with simple algebra the Hamiltonian shown in Eq (4.13).

Appendix B

Another Way of Evaluating The Matrix Element

Now, we are going to discuss another way of evaluating the matrix element which can lead to the same result obtained in Eq.(4.12). This approach involves the construction of the ground state meson wave function and the quark fields which are given in the following form[20]:

$$|M(\mathbf{p},S,S_3)\rangle = \int d^3\mathbf{p}\phi_M(\mathbf{p})\chi(S,S_3)\phi_f\phi_c$$
$$b^+\left(\frac{m_q}{M}\mathbf{P}-\mathbf{p}\right)d^+\left(\frac{m_{\bar{q}}}{M}\mathbf{P}+\mathbf{p}\right)|0\rangle \qquad \text{(B.1)}$$

where $\vec{P},\vec{p}$ denote the C.M. and relative momenta respectively, defined as

$$\vec{p}_1 = \frac{m_{\bar{q}}}{M}\vec{P}+\vec{p} \quad , \quad \vec{p}_2 = \frac{m_q}{M}\vec{P}-\vec{p}$$

and

$$M = m_{\bar{q}} + m_q = m_1 + m_2$$

and the creation and annhilation operators are defined[21] through the equation of Dirac field operator $\psi(x)$

$$\psi(x)=\int d^3\mathbf{p}\left(\frac{m}{E}\right)^{1/2}\sum_{n,c}\left[u_n^c(\mathbf{p})b_{n,c}^f(\mathbf{p})e^{ip.x}+\upsilon_n^c(\mathbf{p})d_{n,c}^{f+}(\mathbf{p})e^{-ip.x}\right] \qquad (B.2)$$

where $\phi_M, \chi(S,S_3), \phi_f, \phi_c$ appeared in Eq(B.1) are the momentum, spin, flavor and color wave fuunctions respectively, the indices n.f and c denoting respectively spin. flavor and color.

The $|0\rangle$ is the vacuum state and it is normalized $\langle 0|0\rangle = 1$. The u and υ appearing in Eq (B.2) are the usual Dirac spinors defined previously. In Eq.(B.2) $(b_{n,c}^{f+}),(d_{n,c}^{f+})$ are the annihilation(creation) operators of a quark and the creation(annhilation) operator of antiquark, with spin-n.flavor-f and color-c. The factor (m/E) is approximitly 1 in the non-relativistic limit so that it will be out of the treatment.

The weak Hamiltonian expressed in Eq (4.13) involves the normal product of quark fields which will be denoted by a pair of colon as shown below. By the use of anticommutation relations of the creation and annhilation operators, the creation operators are made to stand on the left of the matrix element while the annhilation operators stand on the right so that acting on the vacuum state they give zero.

The K^{*0} and π^0 meson states are given by

$$\left|\bar{K}^{*0}(K,S_3=0)\right\rangle = \frac{1}{\sqrt{3}}\int d\bar{k}\,\phi_{\bar{K}^{*0}}(\bar{k})\left[b^{s+}_{j,\downarrow}\left(\frac{m}{M_{\bar{K}^{*0}}}\bar{K}-\bar{k}\right)d^{d+}_{j,\uparrow}\left(\frac{m}{M_{\bar{K}^{*0}}}\bar{K}+\bar{k}\right)\right.$$
$$\left.-b^{s+}_{j,\uparrow}\left(\frac{m}{M_{\bar{K}^{*0}}}\bar{K}-\bar{k}\right)d^{d+}_{j,\downarrow}\left(\frac{m}{M_{\bar{K}^{*0}}}\bar{K}+\bar{k}\right)\right]|0\rangle \tag{B.3}$$

$$\left|\pi^0,S_3=0\right\rangle = \frac{1}{2\sqrt{6}}\int d\bar{p}\,\phi_{\pi^0}(\bar{p})\left[b^{u+}_{i,\downarrow}\left(\frac{m}{M_{\pi^0}}\bar{P}-\bar{p}\right)d^{u+}_{i,\uparrow}\left(\frac{m}{M_{\pi^0}}\bar{P}+\bar{p}\right)\right.$$
$$\left.-b^{u+}_{i,\uparrow}\left(\frac{m}{M_{\pi^0}}\bar{P}-\bar{p}\right)d^{u+}_{i,\downarrow}\left(\frac{m}{M_{\pi^0}}\bar{P}+\bar{p}\right)\right]|0\rangle \tag{B.4}$$

where $\frac{m}{M}=\frac{m}{M_{K^{*0}}}=\frac{m}{M_{\pi^0}}=1/2$ since the u,s and d quarks have approximitly same masses.

The field operators are treated at x=0 , this is due to the translational invariance and by using Eq(B.2) one can calculate the weak hamiltonian which involves four quark fields, the $u, d, \bar{u}, \bar{s}$ as shown below

$$
\begin{aligned}
&:\bar{u}(0)\gamma\gamma_5 s(0).\bar{\bar{d}}(0)\gamma\bar{u}(0)+\bar{u}(0)\gamma_4\gamma_5 s(0).\bar{\bar{s}}(0)\gamma_4\bar{u}(0)\\
&\qquad +\bar{u}(0)\gamma s(0).\bar{\bar{d}}(0)\gamma\gamma_5\bar{u}(0).\bar{u}(0)\gamma_4 s(0).\bar{\bar{d}}(0)\gamma_4\gamma_5\bar{u}(0):\\
&=\sum_{\substack{c,c',n,m\\ n',m'}}\int d\bar{p}_1 d\bar{p}_2 d\bar{k}_1 d\bar{k}_2\Big\{\Big[\bar{u}^u_{c,n}(\bar{p}_2)\gamma\gamma_5\bar{u}^s_{c',m}(\bar{k}_2).\bar{\upsilon}^d_{c,n'}(\bar{k}_1)\gamma\upsilon^u_{c',m'}(\bar{p}_1)\\
&\qquad +\bar{u}^u_{c,n}(\bar{p}_2)\gamma_4\gamma_5 u^s_{c',m}(\bar{k}_2).\bar{\upsilon}^d_{c,n'}(\bar{k}_1)\gamma_4\upsilon^u_{c',m'}(\bar{p}_1)\Big]\\
&\qquad +\Big[u^u_{c,n}(\bar{p}_2)\gamma u^s_{c',m}(\bar{k}_2).\bar{\upsilon}^d_{c,n'}(\bar{k}_1)\gamma\gamma_5\upsilon^u_{c',m'}(\bar{p}_1)\\
&\qquad +\bar{u}^u_{c,n}(\bar{p}_2)\gamma_4 u^s_{c',m}(\bar{k}_2).\bar{\upsilon}^d_{c,n'}(\bar{k}_1)\gamma_4\gamma_5\upsilon^u_{c',m'}(\bar{p}_1)\Big]\Big\}\\
&\qquad \times b^{u+}_{c,n}(\bar{p}_2)b^s_{c',m}(\bar{k}_2)d^d_{c,n'}(\bar{k}_1)d^{u+}_{c',m'}(\bar{p}_1) \qquad \text{(B.5)}
\end{aligned}
$$

where irrelevant terms have been removed. Now, the matrix element can be written in the following form

$$
\begin{aligned}
&\sqrt{2E_{\bar{K}^{*0}}2E_{\pi^0}}\left\langle\pi^0\middle|H^{p.v}_{W}\middle|\bar{K}^{*0}\right\rangle\\
&=\frac{1}{3}\frac{G_F}{\sqrt{2}}\sin\theta_c\cos\theta_c\sqrt{2m_{\bar{K}^{*0}}2m_{\pi^0}}\,\frac{2}{\sqrt{2}(2m)}\varepsilon.\\
&\qquad\sum_{c,c'}\sum_{\substack{n,m\\ n',m'}}\int d\bar{p}d\bar{k}d\bar{p}_1 d\bar{p}_2 d\bar{k}_1 d\bar{k}_2\phi_{\pi^0}(\bar{p})\phi_{\bar{K}^{*0}}(\bar{k})\\
&\qquad\Big\{(\bar{k}_1+\bar{p}_1)-(\bar{p}_2-\bar{k}_2)\\
&\qquad +(\bar{k}_1-\bar{p}_1)-(\bar{p}_2-\bar{k}_2)\Big\}\delta_{nm}\delta_{n'm'}\\
&\qquad\left\langle 0\middle|b^u_{i,\downarrow}\left(\frac{\bar{P}}{2}-\bar{p}\right)d^u_{i,\uparrow}\left(\frac{\bar{P}}{2}+\bar{p}\right)-b^u_{i,\uparrow}\left(\frac{\bar{P}}{2}-\bar{p}\right)d^u_{i,\downarrow}\left(\frac{\bar{P}}{2}+\bar{p}\right)\right.\\
&\qquad :b^{u+}_{c,n}(\bar{p}_2)b^s_{c',m}(\bar{k}_2)d^d_{c,n'}(\bar{k}_1)d^{u+}_{c',m'}(\bar{p}_1):\\
&\qquad\left. b^{s+}_{j,\downarrow}\left(\frac{\bar{K}}{2}-\bar{k}\right)d^{d+}_{j,\uparrow}\left(\frac{\bar{K}}{2}+\bar{k}\right)-b^{s+}_{j,\uparrow}\left(\frac{\bar{K}}{2}-\bar{k}\right)d^{d+}_{j,\downarrow}\left(\frac{\bar{K}}{2}+\bar{k}\right)\middle|0\right\rangle
\end{aligned}
$$

(B.6)

After using the anticommutation relations of the above operators, the integral has four delta functions and we add two more delta functions as a result of using Fourier transformation of the momentum space into configuration space so that we have the expression

$$
\begin{aligned}
&3(2\pi)^{-6}\int d\bar{p}d\bar{k}d\bar{p}_1 d\bar{p}_2 dd\bar{k}_1 d\bar{k}_2 d\bar{r}_1 d\bar{r}_2 d\bar{r}_1' d\bar{r}_2' \,\phi^*_{\pi^0}(\bar{r}_1')\phi_{\bar{K}^{*0}}(\bar{r}_1)e^{-i(\bar{p}_1.\bar{r}_1+\bar{p}_2.\bar{r}_2')}e^{i(\bar{k}_1.\bar{r}_1+\bar{k}_2.\bar{r}_2)} \\
&\left\{\left[(\bar{k}_1+\bar{p}_1)-(\bar{p}_2+\bar{k}_2)\right]+\left[(\bar{k}_1-\bar{p}_1)-(\bar{p}_2-\bar{k}_2)\right]\right\}\delta\left(\frac{\bar{P}}{2}-\bar{p}-\bar{p}_2\right) \\
&\delta\left(\bar{k}_2-\frac{\bar{K}}{2}+\bar{k}\right)\delta\left(\frac{\bar{P}}{2}+\bar{p}-\bar{p}_1\right)\delta\left(\bar{k}_1-\frac{\bar{K}}{2}-\bar{k}\right)\delta(\bar{r}_1+\bar{r}_2)\delta(\bar{r}_1'+\bar{r}_2') \\
&\left\{(\delta_{m\downarrow})^2(\delta_{m'\uparrow})^2-(\delta_{m\uparrow})^2(\delta_{m'\downarrow})^2\right\} \qquad \text{(B.7)}
\end{aligned}
$$

Removing all delta functions will result in the following reduced integral which is contracted further because the first term is just zero since it changes sign as we let

$$
\begin{aligned}
\bar{r}_1 \to -\bar{r}_1 \quad &\text{and} \quad \bar{r}_1' \to -\bar{r}_1' \\
\bar{p} \to -\bar{p} \quad &\text{and} \quad \bar{k} \to -\bar{k}
\end{aligned}
$$

Now, the integral becomes

$$
\begin{aligned}
&(2\pi)^3\left(\bar{K}_{\bar{K}^{*0}}-\bar{P}_{\pi^0}\right)\int d\bar{p}d\bar{k}d\bar{r}_1 d\bar{r}_2'\phi^*_{\pi^0}(\bar{r}_2')\phi_{\bar{K}^{*0}}(\bar{r}_1)e^{2i\bar{p}.\bar{r}_2'}e^{2i\bar{k}.\bar{r}_1} \\
&=\left(\bar{K}_{\bar{K}^{*0}}-\bar{P}_{\pi^0}\right)\phi^*_{\pi^0}(0)\phi_{\bar{K}^{*0}}(0) \qquad \text{(B.8)}
\end{aligned}
$$

Thus the matrix element has this final form

$$M = -\frac{G_F}{\sqrt{2}} \sin\theta_c \cos\theta_c \sqrt{2m_{\overline{K}^{*0}} 2m_{\pi^0}} \frac{2}{\sqrt{2}(2m)}$$
$$\times \vec{\varepsilon}.\left(\vec{K}_{\overline{K}^{*0}} - \vec{P}_{\pi^0}\right) \phi^*_{\pi^0}(0) \phi_{\overline{K}^{*0}}(0) \qquad (B.9)$$

which is just the same result obtained before.

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